PROJECT WHIRLWIND

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FOREWORD

Project Whirlwind

Project Whirlwind at the Massachusetts Institute of Technology Digital Computer Laboratory is sponsored by the Office of Naval Research under Contract N5ori60. The objectives of the Project are (1) the application of an electronic digital computer of large capacity and very high speed (Whirlwind I) to problems in mathematics, science, engineering, simulation, and control, and (2) the study and development of component reliability in Whirlwind I.

The Whirlwind I Computer

Whirlwind I is of the high-speed electronic digital type, in which quantities are represented as discrete numbers, and complex problems are solved by the repeated use of fundamental arithmetic and logical (i.e., control or selection) operations. Computations are executed by fractional-microsecond pulses in electronic circuits, of which the principal ones are (1) the flip-flop, a circuit containing two vacuum tubes so connected that one tube or the other is conducting, but not both; (2) the gate or coincidence circuit; (3) the magnetic-core memory, in which binary digits are stored as one of two directions of magnetic flux within ferro-magnetic cores.

Whirlwind I uses numbers of 16 binary digits (equivalent to about 5 decimal digits). This length was selected to limit the machine to a practical size, but it permits the computation of many simulation problems. Calculations requiring greater number length are handled by the use of multiple-length numbers. Rapid-access magnetic-core memory has a capacity of 32,768 binary digits. Present speed of the computer is 40,000 single-address operations per second, equivalent to about 20,000 multiplications per second. This speed is higher than general scientific computation demands at the present state of the art, but is needed for control and simulation studies.

1. QUARTERLY REVIEW AND ABSTRACT

During the past quarter 46 problems made use of the computer time allocated to the Scientific and Engineering Computation (S&EC) Group. A procedure was developed to eliminate most of the need for manual intervention in reading in the tapes for different problems during a computing period; new routines for automatic curve plotting were introduced into the comprehensive system of service routines.

Two simulated computers were developed, a three-address computer (TAC) and a single-address computer (SAC), which is a modification of the Summer Session Computer described in Summary Report No. 35.

Work on the central computer by the systems group included modernization of the arithmetic-element control, reorganization of the power-supply control for a large portion of the terminal equipment, and equipment changes to facilitate trouble shooting. The addition of two drum groups of auxiliary storage has increased the capacity from about 31,000 to about 36,000 registers.

Life tests involving tungsten-nickel A31 Cathaloy cathode sleeves, being run on 5687's and some special GE Z-2177's, show that there is more interface impedance present on the conducting sections than on the cutoff sections. This is the reverse of experience on active alloys in which silicon is the impurity.

Of the 35 special programs given by MIT during the summer of 1954, two were directly concerned with digital computation, and several others were of interest in the field of computer application. These courses, of 1 or 2 weeks duration, were largely for the benefit of people from business and industry.

2. MATHEMATICS, CODING, AND APPLICATIONS

2.1 Introduction

During the period covered by this report 46 problems made use of the computer time allotted to the Scientific and Engineering Computation (S&EC) Group. Progress reports as submitted by the various programmers are presented in numerical order in Section 2.2. Of these, eighteen (185, 190, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 204, 205, 206, 208, 209, and 211) represent new problems that are being described for the first time. Thirteen problems (147, 149, 169, 174, 175, 185, 190, 192, 196, 197, 198, 202, and 209) have been completed. One problem (143), previously reported as completed, actually continued into the present quarter.

Two simulated computers were developed for use in the 2-week summer session course "Digital Computers: Business Applications." The first, a three-address computer called TAC, is described in Section 5.2 of this report. The other, a single-address computer called SAC, is a modification of the Summer Session (SS) Computer described in Section 5.4 of Summary Report No. 35. The changes are discussed in detail under Problem 196 in Section 2.2 of this report.

A procedure has been developed to eliminate most of the need for manual intervention in reading in the tapes for different problems during a computing period. The procedure is described under Problem 100. It makes use of a specially prepared "director" tape which contains the necessary information and which communicates with the computer through a separate input reader.

New routines for automatic curve plotting have been introduced into the comprehensive system of service routines. A description of these routines will also be found under Problem 100.

2.2 Problems Being Solved

100 COMPREHENSIVE SYSTEM OF SERVICE ROUTINES

The comprehensive system of service routines has been developed by the Scientific and Engineering Computation (S&EC) Group to simplify the process of coding for WWI. The system now in use, called CS II, was described in Summary Reports No. 36, 37, and 38.

Since the reader will find references in some of the reports below to the number system used in CS II, the following brief description is included here for the reader's convenience.

(m, n) numbers shall mean numbers which are of the form $z = x \cdot 2^y$ where x is an m-binary-digit number and y is an n-binary-digit number. For example, (24, 6) signifies a two-register floating-point system dealing with numbers of 24 significant binary digits (roughly seven decimal digits) with magnitudes between 2^{63} and 2^{-64}.

Arithmetic involving these (m, n) numbers is carried out by means of (m, n) interpretive subroutines. These subroutines enable the programmer to write coded programs using (m, n) numbers as easily as, or even more easily than, he might write programs in the single-length fixed-point (15, 0) number system which is built into Whirlwind I.

Director Tapes

During normal operation of the comprehensive system of utility programs a programmer submits a performance request for each group of programs to be run on the computer. The performance request contains a complete description of the run.

A set of standard instructions for use on the performance request has been defined. Included among these are the following:

e,	Press the erase button
fb100-0-0,	Place the designated tape in the input device (generally the photoelectric tape reader).
ri,	Press the readin button. (This reads in a tape placed in the input device and erases core memory if the erase button has previously been pushed.)

The tapes involved in a performance request are prepared and assembled by the Tape Preparation Room. Actual computer operation is performed by trained operators in accordance with the instructions contained on the performance request.

A partial mechanization of this process has been achieved with the introduction of director tapes. A director tape for a particular computer run is a punched paper tape which is obtained by typing the performance request for the run on a Flexowriter.

The comprehensive system of utility routines has been modified so that the majority of computer runs can be made under the control of a director tape. Initially the director tape and the run tapes (spliced together in the proper order) will be placed in separate input devices. The complete run will require only a single pushing of the readin button.

The introduction of director tapes should lead to a more efficient use of computer time with less possibility of operator error.

Additional Automatic Output Features

Ten routines (five interpreted and five noninterpreted) have been written for use in automatic curve plotting.

Two different "formats" are available for prescribing the ranges of the variables to be plotted: (i)FOC 1, for use when the range of variables involved is symmetric, and (i)FOC 2, for use when the range of variables is listed as program parameters.

Field	Description	Problem Number	% WWI Time	Source
Aeronautical Engineering	Construction and testing of a delta-wing flutter model	166 C.	1.86	MIT
	Transient temperature of a box-type beam	179 C.	5	MIT
	Blast response of aircraft	183 D.	3.88	MIT
Chemical Engineering	Transient effects in distillation	*167 D.	5.59	MIT
Electrical Engineering	Switching-circuit design	*169 B.	0.15	MIT
	Crosscorrelation of blast-furnace input-output data	180 B.	0.10	MIT
Geology & Geophysics	Geophysical data analysis	106 C.	2.09	MIT
	Interpretation of earth-surface resistivity measurements	*123 C.	0.54	MIT
	Frequency and phase spectrum analysis of seismograms	192 D.	0.17	MIT
Instrumentation Lab	Servo response to a cosine pulse	211 C.	0.01	MIT
Lincoln Lab	Tracking response characteristics of the human operator	186 C.	0.30	MIT
	Eigenvalue problem for propagation of electromagnetic waves	193 C.	0.50	MIT
	A study of recurrent events	200 C.	0.57	MIT
	Calculation of vertical antenna coverage skeleton	202 C.	0.17	MIT
	Digital methods of detecting signal from noise	149 C.	0.06	MIT
Mechanical Engineering	Flow of compressible fluids (aerothermopressor)	120 D.	2.26	MIT
	Laminar boundary layer of a steady compressible flow in the entrance region of a tube	199 C.	0.99	MIT
Meteorology	Synoptic climatology	155 D.	3.68	MIT
	A scale of turbulence	185 B.	0.48	MIT
Physics	Vibrational frequency spectrum of a copper crystal	143 D.	0.41	MIT
	Energy bands in crystals	147 C.	10.69	MIT
	Use of water storage in a hydroelectric system to minimize the expected operating cost	159 D.	9.01	MIT
	Overlap integrals of molecular and crystal physics	*172 B.	1.00	MIT
	Tight-binding calculations in crystals	174 C.	0.05	MIT
	Impurity levels in crystals	175 C.	4.43	MIT
	Scattering electrons from hydrogen	*184 D.	3.32	MIT
	Coulomb wave functions	*122 B.	0.46	MIT
	Zeeman and Stark effect in positronium	190 D.	0.71	MIT
	An augmented plane-wave method as applied to sodium	194 B.	5.37	MIT
	Study of the ammonia molecule	201 C.	1.05	MIT
	Exchange integrals between real Slater orbitals	204 C.	0.04	Univ. of Chicago
	Electron lattice interaction in solids	205 C.	1.11	MIT
	Electronic energies of the molecule H_2^-	206 A.	0.60	MIT
Servomechanisms Lab	Autocorrelation and Fourier transform calculation	107 C.	0.46	MIT
	Subroutine for the numerically controlled milling machine	132 C.	0.43	MIT
	Data-reduction program; polynomial fitting	126 C.	2.46	MIT
Dynamic Analysis & Control Lab	Interceptor flight-control problem	208 C.	0.08	MIT
Miscellaneous	Comprehensive system of service routines	100	10.83	MIT
	Special problems (staff training, etc.)	131	0.70	MIT
	Library of subroutines	141	0.81	MIT
	Course 6.537: digital computer application practice	173	1.07	MIT
	Summer session: single-address computer	196	6.41	MIT
	Summer session: three-address computer	197	10.10	MIT
	Student problems coded for SAC and TAC	198	4.24	MIT
	Numerical solution of homogeneous linear differential equations with quadratic polynomial coefficients	209 A.	0.17	Camb. Univ. England
	Intestinal motility	195 C.	0.39	Mass. Mem. Hospitals

Table 2-I. Current Problems Arranged According to Field of Application
(* MIT Project on Machine Methods of Computation)

Mathematical Problem	Procedure	Problem Number
1. Matrix algebra and equations		
Evaluation of elements and roots of a third-order secular determinant	Standard analytical solution	143 D.
Solution of a matrix equation	Iteration using Crout's method and matrix algebra	166 C.
Eigenvalues of a matrix	Diagonalization	174 C.
Inversion	Diagonalization	175 C.
Eigenvalues	Diagonalization	190 D.
Eigenvalues	Iteration and diagonalization	201 C.
Eigenvalues	Diagonalization	205 C.
2. Complex algebra		
Complex roots and function evaluation	Iteration	193 C.
3. Ordinary differential equations		
Fourteen nonlinear first order	Gill's method	208 C.
Linear wiht quadratic polynomial coefficients	Taylor series	209 A.
Seven nonlinear first order	Fourth-order Runge-Kutta	120 D.
Second-order linear with variable coefficients	Gauss-Jackson forward integration formula	147 C.
Set of nonlinear first-order equations	Second-order Runge-Kutta	*167 D.
Two second order	Extrapolation by differences	183 D.
System	Gill's method	199 C.
Second-order linear	Power series	211 C.
4. Partial differential equations		
Second-order parabolic	Explicit finite difference approximation	179 C.
5. Integration		
Auto-, crosscorrelation and Fourier transform	Simpson's rule	107 C.
Integral evaluation	Six-point recursion formula	122 B.
Integral evaluation	Trapezoidal rule	*123 C.
Numerical integration	Trapezoidal rule	159 D.
Overlap integrals	Evaluation of analytic forms	*172 B.
Fourier transform of the crosscorrelation of two functions	Simpson's rule	180 B.
Two-dimensional integrals	Simpson's rule	*184 D.
Calculation of transfer functions	Simpson's rule	186 C.
Autocorrelation and Fourier transform	Simpson's rule	195 C.
Integral evaluation	Gaussian quadratic	204 C.
Integral evaluation	Evaluation of analytic forms	206 A.
6. Statistics		
Detecting signal from noise	Weighted counting	149 C.
Multiple time series	Prediction by linear operators	106 C.
Calculation of the coefficients of a multiple regression system	Inner products	155 D.
Correlation of eddy velocities	Tukey's analytic filter	185 B.
Phase and frequency spectra	Simpson's rule and Tukey's method of smoothing	192 D.
Study of recurrent events	Success run and density test	200 C.
7. Data-reduction program	Polynomial fitting, etc.	126 C.
8. Network minimization	Boolean algebra	*169 B.
9. Transcendental equations		
System	Recursion	202 C.
10. Group theory		
Generation of projection operators	Machine generation	194 B.

Table 2-II. Current Problems Arranged According to the Mathematics Involved
(* MIT Project on Machine Methods of Computation)

Point-by-point curve plotting is done by using the instruction (i)SOC a1. The abscissa of the point is contained in register a1, and the ordinate of the point is assumed to be contained in the AC (or the MRA).

Calibrated axes can be plotted on the scope by using the instruction (i)SCX. The coordinates of the origin and the calibration increments are listed as program parameters. Uncalibrated axes can be plotted on the scope by using the instruction (i)SUX. The coordinates of the origin are listed as program parameters.

106 MIT SEISMIC PROJECT

As discussed in various previous reports, Problem 106 is concerned with the investigation of the use of statistical-analysis techniques in seismic-record interpretations, and in particular in the separation of "reflections" from background interference on these records. A more complete description of the problem and the approaches used is contained in "Detection of Reflections on Seismic Records by Linear Operators" (Wadsworth, Robinson, Bryan, and Hurley--Geophysics, Vol. 18, No. 3, July 1953).

Because of the multiplicity of directions in which research on the above problem has gone recently, we shall describe the computational aspects of Problem 106 in two ways, first as they relate to these directions and secondly according to the mathematical techniques they represent. We shall then describe recent results and finally indicate the probable course of future research.

According to the types of research they represent, our computations fall into the following categories:

1. Computations designed to test criteria for automatic selection of optimum linear operators or linear filters;
2. Computations designed to test techniques for probing the statistical and frequency behavior of a given input seismogram;
3. Computations aiding the development of the relationships between discrete linear operators and linear electric filters;
4. Computations designed to simulate and study various statistical noise models;
5. Computations for relating results from 1 and 2 with geologic structure.

The types of computations involved in these categories are varied and overlap considerably. They include: formation of auto- and crosscorrelation matrices, matrix multiplication, eigenroot and eigenvector determination, discrete convolution, formation of auto- and crosscorrelations by several different formulas, power-spectral estimates and amplitude and phase estimates according to various formulas, a variety of linear and quadratic mixing techniques, numerical differentiation and integration, frequency-distribution computation, series generation by moving summation, and others.

Some of the specific results accomplished during the past quarterly period include: the successful testing and use of new criteria mentioned under 1, above; the development and successful testing of new probing techniques under 2, above; the initiation of a noise-generation and study project; and the beginning of direct structural interpretation programs.

Our plans for the future are to pursue research as indicated under all five categories above, probably putting special emphasis on the linear operator - linear filter relationship and statistical noise models, categories 3 and 4.

107 (a) AUTOCORRELATION AND (b) FOURIER TRANSFORM INTEGRAL EVALUATION

Routines developed under this problem by D. T. Ross of the MIT Servomechanisms Laboratory have been used by F. Raichlen of the MIT Hydrodynamics Laboratory in the investigation of turbulent-velocity fluctuations in open-channel flow measured by means of a Pitot-tube pressure-cell combination.

Previously the autocorrelation curves obtained from the Digital Computer Laboratory had a delay-time spacing of 0.0015 second. This spacing didn't give sufficient information in the neighborhood of $\tau = 0$. Therefore 1000 points were taken from these records with a time spacing of 0.000303 second, thus giving more information about the higher frequencies present in the turbulent-velocity fluctuations.

The autocorrelation curves and mean-intensity spectra previously obtained were analyzed to determine the longitudinal scale of turbulence, L_x, a length which is associated with the average eddy size.

Since L_x is defined as $L_x \equiv U\int_0^\infty R(\tau)d\tau$,
where U = mean local velocity, and
$R(\tau)$ = correlation function,

it would seem that the easiest way to arrive at these values would simply be to take the area under the autocorrelation curve between the curve and the line of zero correlation. Because of the natural frequency of the Pitot-tube pressure-cell combination, the autocorrelation curve received from the Digital Computer Laboratory had periodicities superimposed on the autocorrelation curves for the random fluctuations. It would have been very difficult to average out these periodicities, with good precision, to arrive at the random part of the curve and hence the area between the random curve and the line of zero correlation. Therefore, an alternate method was devised whereby the theoretical curve for the mean-intensity spectrum,

$$W(f) = \frac{4L_x/U}{1 + (2\pi f L_x/U)^2},$$

was obtained from assuming the form of the autocorrelation curve, $R(\tau) = e^{-\tau U/L_x}$. By fitting the former curve to the experimentally determined spectra, a series of values of L_τ and L_x (since $L_x = UL_\tau$) for different depths was determined.

The problem then was to determine values of L_x directly from the autocorrelation curves to check the L_x values obtained from the mean-intensity spectra. Basically this was done by realizing that the area under the mean-intensity spectrum and the $\tau = 0$ ordinate on the autocorrelation curve were the mean square of the turbulent-velocity fluctuations. By determining the percentage of the area in the power spectrum representing periodicities and reducing the $\tau = 0$ ordinate of the autocorrelation curve by this percentage, a starting point for the theoretical autocorrelation curve was obtained. Values of L_τ and hence L_x were determined by fitting the curve $e^{-\tau U/L_x}$ to the data, using as the zero ordinate the one explained previously. The results obtained have been found to agree very well with results of other investigators.

120 THERMODYNAMIC AND DYNAMIC EFFECTS OF WATER INJECTION INTO HIGH-TEMPERATURE, HIGH-VELOCITY GAS STREAMS

This problem is connected with the development of a potential gas-turbine component, called an "aerothermopressor," in which a net rise in stagnation pressure of a hot gas stream is brought about by the evaporation of liquid (water) injected into a high-velocity region of the flow. The concepts underlying the operation of the aerothermopressor are an outgrowth of comparatively recent work in the field of gas dynamics, and its intended function in the gas-turbine cycle is analogous to that of the condenser in a steam power plant. Further descriptions of this device may be found in earlier reports, beginning with Summary Report No. 32, Fourth Quarter 1952.

Because of the complexity of the thermodynamic and dynamic processes which occur within the aerothermopressor involving the simultaneous acceleration and evaporation of liquid carried by an accelerating compressible flow, any meaningful theoretical analysis of the problem currently being treated by Whirlwind I involves the simultaneous solution of seven nonlinear, ordinary differential equations.

During the past 15 months, a simple Euler method type of numerical integration was used satisfactorily for computing the performance of constant-area aerothermopressors under various operating conditions. Recently, however, it was found that such simple procedures were entirely inadequate for treating aerothermopressors of varying cross-sectional area, and the resulting truncation errors led to results which were difficult to interpret. The work carried out during the past quarter was devoted almost entirely to the development of a more elaborate numerical procedure which would eliminate serious truncation error and at the same time allow the use of reasonable increment sizes so that the computation time would not be prohibitive.

Two basic methods were studied: (1) a forward-integration scheme involving backward differences in the first derivatives of the dependent variables, and (2) the fourth-order Runge-Kutta method. A third scheme, involving forward and successive integration, was held in abeyance pending results from the aforementioned methods. Detailed studies were made with various increment sizes for a typical case, and it was found that the more elaborate procedures did indeed eliminate the aberrations brought about by truncation error with the Euler method. The Runge-Kutta method proved to be superior from points of view of speed, accuracy, and convenience, since no special starting procedure is required. For an accuracy of about 0.3% in the pressure, the Runge-Kutta method was about six times as fast as the Euler method.

The importance of the variable-area aerothermopressor process has made it mandatory to incorporate the more accurate numerical procedure into the analysis. A new program is accordingly being written which, in addition to using the Runge-Kutta method, will exploit all the latest facilities of the comprehensive conversion system of Whirlwind I. Since the writing of this program will take considerable time, it is planned to continue computations of aerothermopressor performance to a limited extent using the composite program which was used for testing the various numerical procedures. This program suffers primarily from inefficiency, since its basic logic was never intended for this purpose, nor was the auxiliary magnetic drum available at the time it was written.

The aerothermopressor development program is being carried out at MIT under the sponsorship of the Office of Naval Research and is directed by Professor Ascher H. Shapiro of the Department of Mechanical Engineering. The theoretical aspects of the problem treated by Whirlwind I are being carried out by Dr. Bruce D. Gavril.

122 COULOMB WAVE FUNCTIONS

Work was continued by A. Temkin of the MIT Physics Department on the Coulomb wave functions making use of their integral representation. It will be recalled that the regular solution is given by:

$$F_L(\eta_1\rho) = C_L(\eta)\rho^{L+1}\int_0^\infty \frac{\cos(\rho\tanh\xi - 2\eta\xi)}{(\cosh\xi)^{2L+2}}\,d\xi$$

$$C_L(\eta) = \sqrt{\frac{1-e^{-2\pi\eta}}{2\pi\eta}}\left(\frac{1}{2^L}\right)\frac{1}{\sqrt{(1^2+\eta^2)\cdots(L^2+\eta^2)}}$$

A six-point recursion formula is being used for the integration, and our solution was tested against some known results. We found that the accuracy increased as we decreased the mesh size, but that extending the range of integration beyond a reasonable point did practically nothing. Some other considerations also give additional accuracy. Namely, near the origin the function tanh ξ varies roughly as ξ, whereas beyond $\xi = 1$, tanh ξ is practically constant. Thus the frequency of oscillations of the integrand around the origin is determined by $|\rho - 2\eta|$, whereas further out it is determined only by 2η. Since the mesh size should be inversely proportional to the frequency, we test to see if $|2\eta - \rho| > 2\eta$, and if it is we use the mesh size corresponding to it in the region $\xi = 0 \rightarrow 1$.

We are now testing for various values of the parameters to make sure that the routine can handle them all. When this is done, we shall code the irregular solution and the first derivatives of both. We hope to be done with the coding by the end of the next quarter.

123 EARTH RESISTIVITY INTERPRETATION

This problem is concerned with a means of analyzing earth-resistivity data. The procedure is to fit a theoretically derived Slichter kernel to an observed kernel by systematically relaxing the values of resistivities and thicknesses used in the theoretical kernel (see Summary Report No. 37).

The program for this analysis is being modified in an effort to find, empirically, some optimum sequence of parameter-change multipliers. It was found that, because of an error in the program last used, the values of these parameters were increased only once, from 0.05 to 0.1, after the 20th iteration. In addition, it was found that the calculation of exponentials was not as accurate as desired for large arguments, and this is being improved.

Several more kernels have been run. The usual behavior has been convergence for a short while, and then rapid divergence, although one example converged rapidly and continued making small oscillations about the proper solution.

A hand check of the final result of the 91-step analysis described in Summary Report No. 38 indicates that there is some question as to the validity of the final result, since the actual error is greater than that calculated by WWI and is, in fact, slightly larger than the step which gave the correct solution. The reason for this lies in the program and has not been positively located as yet.

This work is being carried out by K. Vozoff of the Department of Geology and Geophysics under the sponsorship of the Project for Machine Methods of Computation and Numerical Analysis.

126 A DATA-REDUCTION PROGRAM

Problem 126 is a very large data-reduction program for use in the Servomechanisms Laboratory. The over-all problem is composed of many component sections which will be developed separately and then combined at a later date. Thus far, efforts have been focused on the development of utility-type programs. These programs, which have been described in previous Quarterly Reports, include a fully automatic program to fit polynomials to arbitrary empirical functions, a general-purpose Lagrange interpolation program, and a flexible and fairly elaborate post-mortem routine. The programs are being developed by Douglas T. Ross and William M. Wolf with the assistance of Miss Dorothy A. Hamilton, Servomechanisms Laboratory staff members.

Three versions of the complete data-reduction program, each version an extension of the previous one, have been tested during this period. Most of the tests have been run using the mistake diagnosis routine (MDR) to extract over 30 intermediate results for accuracy and logical checks. Since the optimum settings for certain parameters are unknown, the facilities of the MDR are nearly indispensable, and the routine use of the MDR has aided the development of these programs considerably. The testing of the third version, called the basic data-reduction program, is nearly complete. When this basic version is complete the first two developmental versions will be discarded, and further elaborations of the basic version will be written.

Work is progressing on another phase of this problem, using auxiliary in-out equipment both as an integral part of the data-reduction programs for monitoring purposes and as an aid to experimenting with improved computational techniques. Intervention registers and scopes will constitute the working parts of this system with efforts being made to provide the flexibility normally associated only with analog computers. Present work is concerned with the development of routines to decode intervention-register inputs and to utilize various specialized scope outputs. After merging these routines with the data-reduction programs, an elaborate logging scheme, using magnetic-tape output, will be written for permanent records of all operations and interventions during an experimental run.

131 SPECIAL PROBLEMS (STAFF TRAINING, DEMONSTRATIONS, ETC.)

This problem has been set aside to account for the WWI time expended in demonstrating the computer to visitors at the Digital Computer Laboratory, in developing routines to be used in these demonstrations, and in testing routines written as part of the training of new staff members.

During the past 3 months, 15 groups visited the Laboratory. The affiliations of some of the larger groups are given in Section 5.3.

132 SUBROUTINES FOR THE NUMERICALLY CONTROLLED MILLING MACHINE

During the first part of this quarter, debugging of a program for preparing milling-machine tape for airfoil templates was completed by J. H. Runyon of the MIT Servomechanisms Laboratory. Much of the remainder of the programming and computer time was devoted to the development of programs for checking milling-machine tapes. These programs are based on a library subroutine for reading, decoding, and detecting errors in the tapes. The programs are for obtaining printouts of decoded milling-machine orders, cumulative totals of orders, and indications of errors. Several tapes containing, in all, about 7500 milling-machine blocks have been successfully checked.

141 S&EC SUBROUTINE STUDY

Several subroutines in the library have been replaced or modified. These are:

LSR FU 1 Exponential Subroutine. New version is 55 registers long and uses one double-length or one buffer register for temporary storage.[1] The first eight terms of the power series for exp x are used to evaluate the function. The subroutine can be used with any (30-j, j) PA.

LSR FU 2 Square-Root Subroutine. Two versions are now in the library. The first, LSR FU 2a, is 34 registers long and uses two double-length or two buffer registers for temporary storage.[1] The square root is formed by Newton's method. Iteration continues until the relative error between the last two iterates is less than 2^{-13}. This program uses (24, 6) arithmetic. The second subroutine, LSR FU 2b, is 27 registers long, with the same temporary storage requirement as FU 2a. In this subroutine, the square root is also found by Newton's method, going through four iterations for all positive numbers. This subroutine can be used with any (30-j, j) PA. In both subroutines, if x is ± 0, control is transferred back to main program with x in MRA. If x is less than -0, the computer stops.

LSR FU 5 Hyperbolic Functions. This subroutine calculates sinh x or cosh x according to where entered. The new version is 89 registers long and uses one double-length or one buffer register for temporary storage.[1]

LSR MA 4 Symmetric Matrix Diagonalization. This is a shorter and faster version of the program described in Summary Report No. 35, pp. 19-21.

1. Specified by preset parameter.

The auxiliary buffer routine (SP 1) has been discarded, since it is no longer useful. Two subroutines have been added to the library:

LSR SP 3 30 x 30 Divide. With a dividend of the form $a + b \cdot 2^{-15}$ and a divisor $c + d \cdot 2^{-15}$, this routine calculates and stores the first two terms of the binomial series

$$\left(\frac{a}{c}\right) + \frac{1}{c}\left(b - \frac{da}{c}\right)2^{-15} + \frac{1}{c}\left(- \frac{bd}{c} + \frac{ad^2}{c^2}\right)2^{-30}.$$

Because of roundoff in the subroutine and truncation in the series, there will be a maximum error affecting the last three binary digits, with a normal error of two digits being affected.

LSR SP 4 30 x 30 Divide. The method of the subroutine is roughly that of the SS divide where the magnitude of the divisor is multiplied by increments, and the results are subtracted from the magnitude of the dividend (or remainder). Smaller and smaller increments are used until $1(x\ 2^{-15})$ x divisor cannot be extracted; then the quotient and dividend are shifted, and the system is run again to give a second quotient register. The results of this subroutine are accurate to the last digit.

143 THE VIBRATIONAL FREQUENCY SPECTRUM OF A COPPER CRYSTAL

The vibrational frequency spectrum for α-iron has been calculated from the secular equation given below, using the atomic-force constants determined by H. Curien.[1] This calculation was performed for 1784 wave vectors uniformly distributed throughout 1/48 of the Brillouin zone. The WWI arithmetic program developed previously for the spectrum calculation of copper (face-centered cubic - F.C.C.) was appropriately modified to yield solutions for the body-centered cubic (B.C.C.) secular determinant of α-iron.

$$| D(q) - I\omega^2 | = 0$$

$$D_{11}(q) = \frac{4}{m}\Big\{2\alpha(1 - \cos u_1 \cos u_2 \cos u_3) + \alpha' \sin^2 u_1 + \beta'(\sin^2 u_2 + \sin^2 u_3) + \alpha''(1 - \cos 2u_2 \cos 2u_3) + \beta''(2 - \cos 2u_1 \cos 2u_2 - \cos 2u_1 \cos 2u_3)\Big\}$$

$$D_{23}(q) = \frac{4}{m}\Big\{\sin u_2 \sin u_3 (2\beta \cos u_1 + 4\gamma'' \cos u_2 \cos u_3)\Big\}$$

$$u_1 = 2\pi a q_1,\ u_2 = 2\pi a q_2,\ u_3 = 2\pi a q_3$$

1. Curien, H., Thesis, L'Université de Paris, 1952.

$$\left|\vec{q}\right| = \frac{1}{\lambda} \qquad \vec{q} = \vec{i}q_1 + \vec{j}q_2 + \vec{k}q_3$$

1st neighbor:	$\alpha = 1.17 \times 10^4$ dynes/cm	$\beta = 1.19 \times 10^4$ dynes/cm
2nd neighbor:	$\alpha' = 1.16 \times 10^4$ dynes/cm	$\beta' = -0.25 \times 10^4$ dynes/cm
3rd neighbor:	$\alpha'' = 0.08 \times 10^4$ dynes/cm	$\beta'' = \gamma'' = 0.26 \times 10^4$ dynes/cm

The composite spectra for the F.C.C. and B.C.C. cases exhibit the same general features. However, it is interesting to note that the two "transverse" branches for the B.C.C. case of α-iron show a somewhat Gaussian-like distribution, whereas for the F.C.C. case of copper these branches have tails only at the low-frequency end, cutting off sharply at the high-frequency end. One suspects that such behavior is caused primarily by the crystal-lattice symmetry rather than by the particular values of atomic-force constants.

This present calculation for a body-centered cubic and the latter calculation for a face-centered cubic[1] contain no assumptions regarding the atomic forces. The force constants are tensor quantities and were experimentally determined from thermal diffuse X-ray scattering experiments. All previous spectrum calculations assumed central forces and considered only first and second nearest neighbor interaction. However, the vibrational spectra calculated from such a model, which is a rather special case, show no essentially different features from the general cases treated by this investigator.

This work was carried out by E. H. Jacobsen of the MIT Physics Department.

147 ENERGY BANDS IN CRYSTALS

Dr. D. Howarth of the MIT Solid State and Molecular Theory Group has calculated the lower energy levels of copper for all typical symmetry points in reciprocal space; he has done this three times, once for each of three different one-electron potentials for copper. The results of these calculations are still being examined, so detailed conclusions cannot yet be drawn. However, the results do show that the energy value of a level depends more strongly than one might suppose upon the choice of the potential. It has also been shown that with the method used calculation of the energy at points of no symmetry in reciprocal space is possible although not practical; such a calculation requires over an hour of computer time.

1. Summary Report No. 38, Digital Computer Laboratory, MIT.

A somewhat surprising result was the disagreement of the present method with Dr. Howarth's previous application of the cellular method to copper.[1] It was found that the relative positions of the twofold and threefold degenerate levels of "d"-like symmetry were interchanged in the two methods, the present method agreeing roughly with those of other workers.[2] Since the present method demands a slightly different form of the potential than does the cellular method, one may assume that this is the origin of the discrepancy. To test this assumption a program was written to calculate these energy levels by the cellular scheme but using the modified potential of the present method. Unfortunately, Dr. Howarth had to return to England before the program had been completely tested.

149 DIGITAL METHODS OF DETECTING SIGNAL FROM NOISE

The study of digital methods of detecting signal in noise has been completed by Dr. G. P. Dinneen of Lincoln Laboratory. A small amount of computer time was used at the beginning of this quarter to complete the study. No significant changes in the results described in Summary Report No. 38 were made. It is interesting to note that in the one case, that of the success-run observer, where partial theoretical results are obtainable, the calculations obtained by the Monte Carlo method on Whirlwind are in excellent agreement with these theoretical results.

155 SYNOPTIC CLIMATOLOGY

During the past quarter, progress has been made, under the direction of Professor T. F. Malone of the MIT Meteorology Department, along six avenues of research which follow directly or indirectly from the statement of the problems given in Summary Report No. 38. Each of these six points will be discussed below.

1. Matrix Programming

The project has advanced to a point where the solution of large-sized matrices is necessary. A program that will handle a symmetrical matrix, using the Crout method, has been written and used extensively during the past few weeks. This is a general program which can readily be adapted to any specific problem. The largest matrix that the routine will handle is determined by the drum storage available for holding the elements of the ma-

1. Howarth, D. J., Proc. Roy. Soc. (London), A 220, 513 (1953)
2. Fletcher, G. C., Proc. Roy. Soc. (London), A 65, 192 (1952)

trix. The program has already been used for matrices up to order 36, and it is expected that it will be used on matrices of the order of 78; the largest matrix that is presently possible appears to be of order 100.

2. Pressure Prediction on New Data

The use of the linear operators derived for pressure prediction was discussed in Report No. 38. Complete pressure predictions have been made for the month of January 1953; this month was not used for the derivation of the linear operator. Twenty-four-hour predictions were made for each of the 91 points on a grid covering most of North America. Analysis of the results of these predictions has not been completed, but a preliminary examination indicates that the accuracy is rather good. This method appears to offer an objective procedure for pressure prediction on a routine basis.

3. Study on Extended Rain Forecasting

Considerable work was done during the past quarter in an attempt to assess the information available in five-day mean circulation patterns concerning the associated five-day mean anomalies of temperature and the five-day accumulation of rainfall. This is essentially an attempt to adapt the techniques embraced in this work to methods of five-day forecasting, as now practiced. In addition to the association of actual weather with circulation patterns, an effort is being made to determine the predictability of the weather elements. It is expected that this work will be completed during the next quarter.

4. Precipitation

Considerable machine time has been used in an attempt to see if it is possible to replace a large number of circulation parameters by a relatively smaller number of parameters which characterize the humidity distribution. The purpose of this work is to predict areal averages of 24-hour accumulation of precipitation. The work has been proceeding by successively more difficult stages, and a final answer on whether the moisture distribution should be introduced explicitly or implicitly, in the form of circulation parameters, has not yet been answered.

5. Four-Dimensional Climatological Model

This work has been proceeding along the lines indicated in Summary Report No. 38. Essentially it involves putting together upper-level circulation patterns with surface patterns and taking account of the time variation in these patterns to predict rainfall and temperature. A good deal of time has been spent in preparing parameter tapes for this work, but no actual runs have been made.

6. Extension of Grid Size

It has seemed worthwhile to investigate the feasibility of extending the present grid which is now bounded by latitudes 30° and 60° North and longitudes 65° and 125° West. The necessary data to do this have been obtained from the U.S. Weather Bureau in Washington, and the work has now advanced far enough to expect that actual machine work will be under way during the next quarter.

159 WATER USE IN A HYDROELECTRIC SYSTEM

The bulk of the work on this problem has been completed and has been written up by J. D. C. Little as part of a doctoral thesis in the MIT Physics Department. A few more calculations may be made during the preparation of papers for publication.

The problem has been: How should the storage water of a hydroelectric system be used when the future river flow is uncertain? Methods currently in use for answering the question are based on analyses of reservoir operation with an assumed future flow. The approach used here is to characterize future flow by probability distributions and determine the water use which minimizes the expected cost of operation.

The Whirlwind computations consist of three parts: the calculation of the probabilities from the historical record of flows, the calculation of tables of best water use, and the calculation of the operation of the system with the historical flows as input.

During the past quarter, the optimization program has been considerably improved, a new scope-output routine has been added to the historical-flows program, and the major production runs have been made. The scope output has been very successful. Originally, it was planned to bring out of the machine on magnetic tape certain results of system operation and then plot representative data by hand. This soon proved impracticable, both because of the amount of magnetic tape used and because of the hand labor required. Therefore, a scope routine was written which displays a set of four (actually, any number) curves on the scope. For a given method of determining water use and for a given year in the historical record, curves were displayed to show the historical flow, recommended storage-water use, required supplemental generation, and reservoir elevation, all as functions of the time through the year. Although too gross to show up fine differences of operation, the curves give an excellent feeling for how the system is behaving macroscopically.

Operation as determined by the expected-value method has been compared to a more conventional operation on a simple model of a hydro system idealized from the Grand Coulee plant on the Columbia River. Differences between various kinds of operation were found to be small. However, the expected-value method gives a sensible stable operation with an average cost somewhat lower than that of conventional operation.

166 CONSTRUCTION AND TESTING OF A DELTA-WING FLUTTER MODEL

The problem and computational procedures of designing a delta-wing flutter model to simulate a given set of flexibility influence coefficients of a full-scale wing were described in Summary Reports 37 and 38 by M. M. Chen and S. I. Gravitz of the MIT Aeroelastic and Structures Research Laboratory.

The entire programming of the iterative procedure for a three-bay structure has been completed and is now in working order. Four basic tapes are involved in the computation. The first one is a Flexo tape which is designed for the direct readin of the given influence coefficients and the first estimates of the stiffnesses of each individual member. The second tape is in binary (556) form and contains the geometry of the system. The third tape is also in binary form and corresponds to the calculation of ΔC's. The influence coefficients are computed on the basis of the first estimates of the stiffnesses, and the differences are printed out. The last tape, in binary form, corresponds to the formulation of ΔX's. If the trial influence coefficients are not close enough to the given ones, corrections must then be made to the trial stiffnesses.

The program works as follows: For the first cycle all four tapes are read into the computer. The preliminary output from each cycle is stored on the drum right over the corresponding output from the previous cycle. Therefore, for each cycle of the iteration, only the last two computational tapes need be repeated.

The last step of the fourth tape makes use of Crout's matrix-inversion routine which solves the set of linear simultaneous algebraic equations to yield the corrections. Unfortunately, the principal difficulty in the iterative procedure arises in this routine; unreasonable answers were obtained because of the ill-conditioned matrix of coefficients. At present satisfactory results for a three-bay model were obtained by a trial and error scheme. The total deviation of the flexibility influence coefficients from the given ones is on the average about 7%, and the error of the influence coefficients of the diagonal element runs from 0.1% to 5.8%.

During this period, a three-bay, 5/32-inch aluminum model was built and tested. The experimental results agreed fairly well with the theoretical influence coefficients, and the average error is about 3%.

167 TRANSIENT EFFECTS IN DISTILLATION

Work has been continued by J. F. O'Donnell of the MIT Chemical Engineering Department on two basic problems which have been described in preceding reports. The results will form a part of his Sc.D. thesis, which is being done under the supervision of Professor E. R. Gilliland. Each of these problems involves transient effects of column holdup in binary batch distillation. The problems are: batch distillation, product takeoff at constant reflux ratio, and transients in continuous distillation.

One new program has been developed and operated successfully on Whirlwind during the past quarter. It solves the equation for the limiting case of product takeoff in binary batch distillation at constant reflux ratio, assuming the holdup in the column is negligibly small.

The basic relationship for this case is:

$$W = \exp\left\{\int_{X_{W,o}}^{X_W} \frac{dX_W}{X_D - X_W}\right\} \qquad (1)$$

where W is the fraction of the charge not yet distilled, X_D is the distillate composition, and X_W is the still-pot composition. From a combination of material and energy balances and equilibrium relationships, X_W can be expressed explicitly in terms of X_D. However, the only way X_D can be obtained from X_W is by iteration. The initial value of X_D corresponding to $X_{W,o}$ is calculated by iteration. Thereafter, X_D is chosen and X_W calculated. The resulting increments in X_W are unevenly spaced. One of the problems was the question of how to carry out the integration.

Initially it was hoped that by using the trapezoidal rule for the integration, solutions of sufficient accuracy could be obtained with reasonable machine time. Two sample cases were run on the machine, each with at least three different increment sizes. Then it was determined that a plot of the result, W, for any value of X_W against h^2, where h is the increment size, was a straight line. This implied that Richardson's h^2 extrapolation might be used.

Briefly, the following procedure is used for the integration. Starting at one point, values of W are calculated using two different increment sizes alternately with one equaling half the other. That is, two steps are taken using the smaller increment size and then one step with the larger. Thus, at that new point there are available, at a single value of X_W, values of W calculated using the two increment sizes. Then the mathematical equivalent of Richardson's h^2 extrapolation is done, giving a corrected value of W. This is printed out.

The results from this procedure were very satisfactory. Less machine time and output were used than would have been necessary if the two calculations had been done separately (because it was not necessary to calculate X_W from X_D again for the larger step). Also less machine time was used than would have been necessary if each case had been done using only one increment size, small enough to insure sufficient accuracy. A check was made each time the extrapolation was carried out to see that it was not too large.

Using this program a series of cases has been solved, giving the product composition as a function of the fraction of the charge distilled. These cases with holdup negligible include all the sets of parameters which had been used previously to obtain solutions for the same operation assuming varying amounts of holdup present. Comparison of the two types of results has been valuable in gaining an understanding of the effects of holdup.

The major part of the work done on transients in continuous distillation has been on the problems of quantitative correlation of the data for the case of a step change in feed composition. Two correlations were developed for the data obtained previously. Then additional solutions were made using Whirlwind to determine the effectiveness of the correlations in predicting results for different parameter combinations. The work showed that the correlations were quite successful for conditions within the range previously studied. However, some extreme sets of parameters gave results that were not predicted by the correlation satisfactorily for engineering purposes. J. I. Jordan has submitted a Master's thesis to the Chemical Engineering Department on this phase of the subject.[1] A considerable improvement in understanding of the situation has been achieved.

Work is continuing on the over-all project. Though some additional data will be taken, the major effort will be on the utilization of available data to reach qualitative and quantitative conclusions.

169 UTILIZING A GENERAL-PURPOSE DIGITAL COMPUTER IN SWITCHING-CIRCUIT DESIGN

As this is the final report on this problem, a brief review of the problem and the results that have been obtained will be given.

The problem undertaken by E. C. Hoy of the MIT Electrical Engineering Department involves the simplification of Boolean functions or switching functions, which are Boolean functions interpreted as representations of series-parallel switching circuits. Because Boolean functions may be interpreted as switching-circuit representation, the problem of their simplification is of more than academic interest. The arbitrary criterion for simplicity is the fewness of independent variable appearances in a function form.[2] In general for a given function there is no unique form of any chosen degree of simplicity. Thus there may be several forms of the greatest degree of simplicity. It is desirable to locate at least one such simplest form with full cognizance of the fact that such a form has been obtained.

A program for simplifying switching functions has been developed based on the ap-

1. Jordan, J. I., Jr., "Transient Conditions in Continuous Distillation Columns," S.M. Thesis, Chemical Engineering Department, MIT (1954).
2. Other criteria may be chosen, e.g., fewness of the number of variable appearances plus terms.

plication of several logical (or Boolean) identities. In the course of the programming, methods for adapting the digital computer to Boolean algebra were developed, including a method for representing the function in storage and methods for performing the three major logical operations.[1] Input and output to the program are made in notations familiar to those working in the switching-circuit field. Delayed output by means of magnetic tape is used to save computer time. The program requires 506 registers of fast storage in addition to the storage required for the statement of the particular problem. At present the program can effectively handle any function of nine variables. A function of any number of variables up to and including 15 can be handled providing its statement does not exceed the storage available. By using auxiliary storage or by sacrificing some of the input and output conveniences, the program can be revised to handle more functions than it does at present. Providing the number of variables does not exceed 15, the number of variables does not directly affect the operating time of the program. Starting with standard normal forms,[2] the program required less than 23 seconds to simplify 26 functions averaging over 10 terms each. However, although the computer simplifies functions much faster (about 100 times faster) and more accurately than people using the same simplification method, the simplification method that has been programmed does not insure simplest solutions and does not indicate the nature of the solution.

Various methods for modifying the existing simplification method were studied, and different approaches to the problem were sought. The most promising of the methods investigated is that proposed by W. V. Quine (see reference below). His method has a proof indicating the nature of the solution.

The answer to the question of whether it is feasible to use a general-purpose digital computer to simplify switching functions is necessarily a subjective one, depending upon the particular engineering problem of the designer and the equipment available to him at reasonable cost.

For a more complete discussion of this work than presented here, one may refer to a forthcoming Electrical Engineering S.M. thesis by E. C. Hoy entitled, "Simplifying Switching Functions Using a General Purpose Digital Computer."

References: Shannon, C. E., "A Symbolic Analysis of Relay and Switching Circuits," Transactions of the AIEE, Vol 57, pp 13-23, 1938

Quine, W. V., "The Problem of Simplifying Truth Functions," American Mathematical Monthly, Vol 59, pp 521-531, 1952

1. See Quarterly Progress Report No. 11 of the Project on Machine Methods of Computation, p. 9.
2. I.e., each variable appears once in each term.

172 OVERLAP INTEGRALS OF MOLECULAR AND CRYSTAL PHYSICS

The routines developed by F. J. Corbató of the MIT Physics Department and described in Summary Report No. 38 have been used to obtain the integrals arising in the tight-binding calculation of graphite. These integral values are the input data for a combined generation/secular-equation program which for a series of reciprocal lattice vectors, $\vec{k}$, generates the dependent matrix elements of the equation,

$$\sum_j H_{ij} X_{jm} = \sum_j S_{ij} X_{jm} E_{mm},$$

and solves for the eigenvalues, E_{mm}, and eigenvectors, X_{jm}. This program was used for several sets of possible integral values. The results revealed that the secular-equation solutions were sensitive to inaccuracies in the S_{ij} matrix and that a greater number of integrals than had been suspected heretofore were required for the necessary accuracy in the S_{ij} matrix elements. Inasmuch as the present generation/secular-equation program is not easily extended to a greater number of initial integral values, the entire program is currently being rewritten.

During the latter part of this quarter, time was spent consolidating and partially generalizing the integral-evaluation program. In addition, on the basis of the experience gained from the present work, several improved subroutines were offered to the subroutine library.

173 COURSE 6.537 -- DIGITAL COMPUTER APPLICATION PRACTICE

Two of the students who were enrolled in the MIT Electrical Engineering course 6.537, entitled "Digital Computer Application Practice," given by Professor C. W. Adams, have continued to work on special problems.

The time spent on Whirlwind by H. C. Kreide has been used to develop a date-demonstration routine as a special problem for course 6.68. The purpose of the demonstration routine is threefold: (1) to supply the day of the week that specific dates fall on; (2) to supply all of the dates in a particular month upon which a given weekday falls; and (3) to supply the date of Easter for any desired year.

The routine is based on the Gregorian Reform and handles all dates between 1600 and 16,000 A.D. Calculations are based on the theory of congruences. Basis for correct dates and weekdays has been Cunningham's "250 Year Calendar." Basis for correct dates of Easter has been the 1954 World Almanac. All tests made on the final routine have yielded correct results (only the years between 1753 and 2100 are given in publications).

As it presently stands, the routine consists of four tapes, the last of which contains the dates about which information is requested. It will correctly answer questions about

dates when given in proper form. A few revisions will be made to allow the information given on Flexowriter tape to be more flexible and to reduce considerably the possibility of misinterpreting it. Also, some revisions will be made in information that is printed out when illegal or nonexisting dates are given the computer. Any further revisions will be to eliminate redundancies. Even without revisions, the program is in good operating order.

The problem considered by S. Petrick was the adaptation of a form of the game "battleship" to regular WWI code. The user of the program decides on how he wishes to place his five "ships" on a 16 x 16 grid. The ships have lengths from two to six squares long and must lie either horizontally or vertically. In addition, no two ships can be adjacent along a line.

The machine picks a square and then consults a stored table to see if a hit has been made. If so, control is transferred to a polishing-off routine. The method used is the following: Consider square x_{ij}. Determine the number of different ways in which a class-two ship can be placed on the board at this time. Repeat this process for class-three, -four, -five, and -six ships. Add the six resulting quotients to give an "index of shootability" for square x_{ij}. Repeat this process for all other squares, and then pick the square whose index is greatest.

At present the routine has been found to work except for the action of the polishing-off routine on a class-six ship. It is hoped that this error can be found and that no other shortcomings of the polishing-off routine will turn up.

Only one case which worked satisfactorily has been tried, and the same distribution of ships was given to several human players. While Whirlwind did not take fewer guesses than the best human guessers (two of 15 had lower scores), its score was much below the average human score.

174 TIGHT-BINDING CALCULATIONS IN CRYSTALS

The unperturbed wave function for a crystal is approximated by a linear combination of atomic functions. A Hamiltonian matrix is set up between these functions; the eigenvalues of this matrix are the energy levels of the electrons. The diagonalization of the Hamiltonian has been done on WWI.

In particular, this program has now been carried out for the nickel metal. This involved the diagonalization of 680 five by five matrices. The matrix elements were calculated in the machine instead of just reading in the matrix elements after computing them by hand. This calculation has now been completed, and the results have been processed to supply a density of states for nickel. This density of states represents the number of energy levels in the nickel metal which are in a given energy range and is important in studies of magnetism and specific heat in solids. The results are now being prepared for publication in The Physical Review by Dr. G. Koster of the MIT Solid State and Molecular Theory Group.

175 IMPURITY LEVELS IN CRYSTALS

The calculation of impurity levels in crystals involves the solution of difference equations which are solved by a Green's function method. This involves finding the inverse of matrices (H-E) where H is the unperturbed Hamiltonian and E is the energy of the impurity level. The inversion is being done on Whirlwind I by Dr. G. Koster of the MIT Solid State and Molecular Theory Group.

The routine for inverting the difference of the Hamiltonian matrix and the energy E has been applied to chromium in order to study the energy levels of impurities in this metal. For each Hamiltonian matrix H, 16 different E values were subtracted from it before inversion. The total number of Hamiltonian matrices which were considered was 220. (The method of determining the energy levels of the impurity atom in the solid from the inverses of (H-E) is described by G. F. Koster and J. C. Slater in Phys. Rev. 95, 1167 (1954).)

A knowledge of the impurity levels in chromium is useful in studying the variation of the saturation magnetic moment in the transition elements as it varies with impurity calculation. The work on this problem, which was to be carried out on Whirlwind, and the results of the computation are now being processed by the method described in the reference above to give the actual levels in the solid.

179 TRANSIENT TEMPERATURE OF A BOX-TYPE BEAM

The time histories of temperature and stress at several points along the periphery of a thin-walled rectangular cross section have been calculated on WWI by L. Schmit of the MIT Aero-Elastic and Structures Research Laboratory. By using an extruded rectangular cross section (of 63X-T5 aluminum alloy), the joint contact admittance problem has been avoided. The heat input is constant with time but varies chordwise. All spanwise effects are neglected. The temperature gradient across the thin wall thickness is assumed to be negligible. The specific heat, thermal conductivity, and weight density are assumed not to vary with temperature. However, the modulus of elasticity and the thermal coefficient of expansion are assumed to vary with temperature. The temperature distribution is carried out by an uncoupled finite-difference procedure. The physical grid employed consists of 54 1/8-inch square elements. A time increment of 0.02 second is selected ($M = \frac{(\Delta S)^2}{a \Delta \tau} = 6.25$). Upon completion of every 50 time cycles the stress distribution is determined on a basis of the current temperature distribution. The stress calculations are based on an extension of the elementary-beam theory to include thermal effects. Both the

temperature and time histories obtained for a heating duration of 25 seconds are to be compared with experimental results.

180 CROSSCORRELATION OF BLAST-FURNACE INPUT-OUTPUT DATA

A combined program which will compute and plot for any given functions a correlation function and its Fourier transform is now being optimized and tested. The program is designed to minimize the tape-preparation time, operating complexity, and total computer time involved in correlating a pair of functions and computing the transform of the correlation function. In addition, the program eliminates all intermediate hand computation and tape preparation, since a single continuous run completes all computation.

A further aid in operating the program is the use of only the oscilloscope camera as the output device; the problem of efficiently scheduling magnetic-tape utilization in running a series of problems is greatly simplified.

The program will handle functions up to 250 values in length and uses the Fourier transform program developed under Problem 171 by D. T. Ross. This program has the feature of automatically reducing spurious Gibbs ripples in the spectra. Both cosine and sine transforms are computed. However, when a function is autocorrelated, only the cosine transform is required, and the program may be stopped after this result is obtained, thus eliminating a lengthy and unnecessary computation.

It is hoped that the availability of such a program in the Laboratory files will materially reduce the programming effort involved in many types of problems, since the processing which this program automatically carries out has very wide applications in scientific and engineering computation. This work was initiated by R. G. Mills for an S. B. thesis in the MIT Electrical Engineering Department.

183 BLAST RESPONSE OF AIRCRAFT

During this past quarter, three new phases of the blast loading problem have been programmed, and test runs have been made on WWI.

The problem, being programmed by Y. Shulman of the MIT Aero-Elastic and Structures Research Laboratory, is to determine the dynamic response of an idealized airplane upon encountering a symmetric blast gust. The airplane is approximated by one fuselage mass and two wing masses connected by two uniform massless beams. The method of solution is, in general, similar to that of the cantilever-beam problem reported earlier.

Unlike the cantilever-beam case studied earlier, the structural characteristics of the uniform beam, which represents the wing structure, are represented by a moment-vs.-curvature curve having a discontinuity in bending moment at the instant that the wing buckles. Subsequent to buckling, the resistance of the beam decreases as its root curvature increases. In view of this consideration, the programming of the problem provides an additional feature: namely, the maximum response of the structure is defined either as the actual peak curvature of the curvature at which the resistance of the beam is zero.

The blast force acting on the wing is formulated according to unsteady aerodynamic theory and lumped at the mass stations. The exponential form of Wagner's function is used, and hence the subroutine for e^x is used in the program.

In addition to the blast force and inertial force, the airplane is also subjected because of motion to damping forces which were neglected in the cantilever-beam case. For one phase of the present problem, the damping force is formulated according to unsteady aerodynamic theory and reduced to a recurrent form. For a second phase, the damping force is assumed quasi-steady, and the numerical-integration procedure is greatly simplified.

Tapes of the above two phases of the problem have been tested and checked with desk solutions. Results of the runs also show that the assumption of quasi-steady damping, though satisfactory in the linear range of the structural characteristics, yields large error in the post-buckling range.

Furthermore, the test runs of the tape also indicate that the machine time needed for the automatic reduction of the time increment is longer than desirable. Consequently, a convergence test has been made for a set of parameters of practical interest without automatic reduction. It was found that for a nondimensional time increment of 0.1 the error in amplitude is within 2%, and the error for the time when the peak occurs is negligible.

A third phase of the problem is to determine the peak blast-gust velocity which will cause a given wing structure to fail at various altitudes and airplane velocities. For a set of parametric values, the problem thus involves a cut-and-try procedure so that the peak response is equal to the given failure curvature. A tape has been prepared with the provision of an automatic increase in the value of the peak blast gust. The tape is, at present, being tested on WWI.

The production runs scheduled for the near future consist of 50 runs for the first tape and 40 runs for the third tape. The machine time needed for the first tape is estimated at about 3 minutes per run. No estimate can be made at this time for the third tape.

184 SCATTERING OF ELECTRONS FROM HYDROGEN

The calculation of the two-dimensional integrals described in Summary Report No. 38 has been largely completed. Each integral corresponds to a probability of a virtual transition to an excited state of the hydrogen atom. These expressions were evaluated for

three values of the energy of the incident electron and five values of the scattering angle. It was possible to perform the integrations analytically when the scattering angle was zero degree. This exact calculation was a check on the numerical procedure, and the two results checked to the desired accuracy (three significant figures).

In the case of the virtual transitions to the continuum states, it had been planned to reduce what were originally three-dimensional integrals to a two-dimensional form and then perform the numerical quadrature, as in the case of the transitions to the discrete states. However, it finally had to be conceded that the approximations that led to this reduction were unsatisfactory, and a program was constructed to perform the original three-dimensional integrals numerically. These terms are also being checked against the results of simpler calculations for zero scattering angle.

This work is being carried out by M. C. Newstein of the MIT Physics Department and will be included in his Ph.D. thesis.

185 A SCALE OF TURBULENCE

The correlation of eddy velocities or fluctuations between two points within a turbulent flow will generally vary inversely with the magnitude of the distance between the points. This offers a method by which eddy size may be defined, the length of the eddy being that separation of the points at which the correlation becomes insignificant.

Atmospheric turbulence covers a wide range of motions with associated length scales varying from large disturbances containing most of the energy of the motion to those smallest eddies which dissipate energy as heat by action against viscosity. J. W. Tukey states that the minimum resolution of fluctuations is fixed by the sampling interval on a stationary stochastic process. An upper limit is set by the length of the sample.

Tukey's analytic filter was applied to seven sets of wind-velocity data. These data were simultaneous observations of wind speed at a height of about 2.5 meters made by four standard Hastings hot thermocouple anemometers with a time constant of 0.25 second. The anemometers were aligned either parallel or normal to the mean wind flow with a separation of 1, 2, and 4 meters between individual units on five of the observations and 3, 6, and 12 meters on the remainder. The sampling periods were from 392 to 510 seconds long. The sampling interval was 1 second.

The analytic filter operates on the sequence

$$x_1, x_2, \cdots x_{i-1}, x_{i+1}, \cdots, x_{N-1}, x_N$$

by forming the following differences:

a. First-order differences of the j-th probe,

$$d_j^{(1)} = (x_{2i} - x_{2i-1}), \qquad i = 1, 2, \ldots, \qquad [N/2]$$

b. Second-order differences for the j-th probe,

$$d_j^{(2)} = \left\{(x_{4i} + x_{4i-1}) - (x_{4i-2} + x_{4i-3})\right\}, \qquad i = 1, 2, \ldots, \quad [N/4]$$

c. Third-order differences for the j-th probe,

$$d_j^{(3)} = \left\{(x_{8i} + \ldots + x_{8i-3}) - (x_{8i-4} + \ldots + x_{8i-7})\right\}, \quad i = 1, 2, \ldots, [N/8]$$

where $[N/2^p]$ represents the integral part of $N/2^p$.

According to Tukey, the sequences of differences as formed above provide series in which maximum spectral power is concentrated near periods of 2^p seconds per cycle, where p is the order of the difference. The filter is only a rough one since it lacks sharp cutoff characteristics. In this problem, only the first three differences are considered.

Other investigators have shown that the sums of squares of the differences defined above are proportional to the variance and energy at their associated frequencies.

Seven parameter tapes were prepared for presentation to Whirlwind using the observed wind-speed reading. A program was then prepared in which the differences previously defined were formed intermediately without printout. For each difference, for each parameter tape, Whirlwind was called upon to provide the following numbers as its contribution to the solution of the problem:

$$\sum_{i=1}^{[N/2^p]} d_{ij}, \quad \sum_{i=1}^{[N/2^p]} d_{ij}d_{ik}, \quad \sum_{i=1}^{[N/2^p]} d_{ij} - d_{ik}, \quad \sum_{i=1}^{[N/2^p]} d_{kj}^2 d_{ik}^2, \quad j, k = 1, 2, 3, 4$$

where d_{ij} is the i-th difference in the sequence of differences for the j-th probe. Thus 630 numbers were printed by Whirlwind.

With the results from Whirlwind, certain statistical tests were made on the data in order to ascertain the degree of validity of the assumption that the d_{ij} are normally distributed with mean of zero and with homogeneous variances for synoptic observations. This was accomplished by considering each difference for each of the seven sampling periods separately as a subject for the following battery of tests. First, the homogeneity of variances was tested by using the test described by Hoel for this purpose. Then an Analysis of Variance was performed to test the hypothesis that the means from each of the four probes were essentially the same, and the Student's "t" Test was applied to show that this mean was not significantly different from zero. Finally, a Chi-square Goodness of Fit Test was applied to the d_{ij} to determine if they could be considered as having a normal distribution with mean zero and a variance equal to the arithmetic mean of the variances

of the four individual probes.

For each of the differences for each sampling period, the correlation coefficients for the six possible combinations of the probes were computed. These coefficients were plotted against distance, and the indicated length at which the coefficient becomes sensibly zero was noted. Thus, an eddy with a predetermined frequency will have a length, or scale of turbulence, associated with it.

In addition to the usual correlation coefficient, the d_{ij} were first squared (by Whirlwind) and then a figure analogous to the correlation coefficient computed and plotted against probe separation. Also, the quantities $\Sigma \mid d_{ij} - d_{ik} \mid$ were plotted against probe separation. The last two plotted figures will be compared to the plotted correlation coefficients to determine the degree of agreement between them.

Production runs have been satisfactorily completed on all seven sets of data. The results have been described in an S.M. thesis written jointly by J. Howcroft and J. Smith for the MIT Meteorology Department.

186 TRACKING RESPONSE CHARACTERISTICS OF THE HUMAN OPERATOR

A series of power-density and cross-power-density spectra has been computed for J. I. Elkind of the Lincoln Laboratory at MIT by WWI. These spectra were computed by taking the Fourier transform of the auto- and crosscorrelation functions of stimulus and response. The spectra that have been computed were obtained from an experiment designed to measure (1) the variability in the response characteristics of particular subjects, and (2) the variability in response characteristics among several subjects.

Only a few of the spectra have been used to obtain human operator transfer functions and noise power-density spectra. Most of the results already computed by WWI have not yet been reduced to this form. The transfer functions that have been obtained show that the human operator acts like a low-pass filter of band width about as great as the band width of the stimulus he is tracking.

190 ZEEMAN AND STARK EFFECT IN POSITRONIUM

Positronium, the transient electron-positron atom, annihilates from S-states, with the transfer of the rest mass energy to the free radiation field. From the $^3S_1$ states the annihilation process yields 3 quanta, from $^1S_0$ states two. The gross energy structure of the atom is the same as that of hydrogen save for a constant factor of 1/2 arising from the reduced mass of the "nucleus." The fine and hyperfine structure is entirely different.

In the n = 2 group of states the fine structure pattern perturbed by magnetic and electric fields has been computed by diagonalizing the Hamiltonian for the system. The electric fields arise experimentally from the motion of the atom perpendicular to the magnetic field in which it is formed. The Hamiltonian matrix is 14 x 14 and has been diagonalized for values of the magnetic field of 500, 1000, 2000, 3000, 4000, and 5000 gausses and for the values of the motional energy of 0, 0.025, 0.25, and 0.5 electron-volt.

The states of n = 2 all have finite lifetimes (2- or 3-quantum annihilation or optical emission), and an approximation to the reciprocal lifetimes of the perturbed states Γ_i^{Total} (i = 1, ..., 14) may be found from the unperturbed reciprocal lifetimes γ_k(k = 1, ..., 14) by computing

$$\Gamma_i^{\text{Total}} = \sum_k (a_{ik})^2 \gamma_k,$$

where a_{ik} is the perturbed eigenvector.

The branching ratio for 2- to 3-quantum annihilation of the atom may be found by forming the partial sums

$$\underset{\text{singlet}}{\Gamma_i = \sum_{k\ \text{singlet only}} (a_{ik})^2 \gamma_k} \quad \text{and} \quad \underset{\text{triplet}}{\Gamma_i = \sum_{j\ \text{triplet only}} (a_{ij})^2 \gamma_j}$$

For each energy eigenvalue computed, these partial reciprocal lifetimes and the total reciprocal lifetime were also found.

The present program of computations is now completed. The study was carried out by H. W. Kendall of the MIT Radioactivity Group. The routines for diagonalizing the Hamiltonian and for computing the partial and total reciprocal lifetimes were developed by Dr. A. Meckler of the MIT Solid State and Molecular Theory Group.

192 FREQUENCY AND PHASE SPECTRUM ANALYSIS OF SEISMOGRAMS

The following constitutes a brief resume of the work accomplished under Problem 192 by William P. Walsh. A more detailed account of the work may be found in the author's Ph.D. thesis submitted to the Department of Geology and Geophysics, 16 August 1954.

Throughout the existence of seismology surprisingly little has been done to ascertain by means of exact analysis the frequency content of recorded seismic motion, in comparison to the theoretical work which has taken place in this regard. The U.S. Coast & Geodetic Survey has, we understand, recently carried out a good deal of work in this direction. Their attention was confined to strong-motion earthquakes, however. Many investigators in the petroleum industry have recently become interested in the frequency analysis of reproducible prospecting records. As far as we know, the aforementioned is all the experimental research that has been projected in this direction in the fields of exploration and earthquake seismology.

In our work we have endeavored to apply the recent findings of Tukey (outlined in our thesis and initial report) to the spectrum analysis of seismograms and to exploit as much as possible MIT's WWI computer to that end.

Our purpose was not only to devise a means of seismogram spectrum analysis but also to apply such analysis in several cases in order to obtain useful information concerning surface-wave dispersion, microseisms, and direct, refracted, and reflected energy of earthquakes and explosions.

The procedure we devised consisted of computing spectra (à la Tukey) from hundreds of successive overlapping intervals and displaying them in the fashion of a contour map -- called here traveling spectra.

Application of the aforementioned process to several earthquake records obtained from the observatory in Weston, Massachusetts, revealed some interesting dispersion curves for both Rayleigh and Love waves over Atlantic and continental paths. Those curves for surface waves which traveled the Atlantic path were strikingly similar, whereas those curves for surface waves traveling over the continent were most dissimilar. This probably indicated a fair degree of lateral inhomogeneity in the continental portion of the earth's crust by way of comparison to the oceanic portion. It appears that exact comparison of our observed curves with models which more clearly delineate the complexities of the earth's crust must await the calculation of theoretical curves obtained from at least a five-layered model (because of difficulties in these calculations only curves of two-layered models have thus far been determined).

Similar techniques were also applied to some microseisms (seismograms recorded at Weston) due to a storm which existed over the Atlantic near the coast of New England. Our analysis revealed the existence of a resonance phenomenon (i.e., the depth of water under the meteorological "low" or cold front dictates the band width and prominent frequencies of the microseism spectrum) and further corroborated the theory of the origin of microseisms proposed by Longuet-Higgins.

Since the analysis of earthquake seismograms suggested the use of traveling spectra as a means of determining the travel time of reflections, several trials were made on three prospecting records where reflections were both visible and invisible. The results were rather encouraging, for all types of reflections were put into evidence quite well.

193 EIGENVALUE PROBLEM FOR PROPAGATION OF ELECTROMAGNETIC WAVES

The field radiated from a Hertzian dipole to points well beyond the horizon over a perfectly reflecting earth through an atmosphere whose index of refraction decreases linearly with height (constant gradients) can be conveniently calculated as an eigenvalue problem. The eigenvalues of the allowed modes are determined by the boundary conditions at

the earth's surface, at the air-vacuum interface at height g, and at heights above the air. The real parts of the eigenvalues determine the phase velocities of the allowed modes, and the imaginary parts give the attenuation rate with horizontal distance around the earth. The earth radius and wavelengths considered, of the order of a few meters or centimeters, enter also into the numerical values of s and g, the index of refraction gradient and air-layer thickness, respectively. The first step, finding the eigenvalues w of the propagation modes consistent with the boundary conditions, leads to the problem of finding the complex roots of the following equation:[1]

$$\frac{h_1(w - sg)}{h_2(w - sg)} = \frac{8s^3}{1 - s^3} e^{-i\pi/3} w^{3/2} \operatorname{expon}\left[i \frac{4}{3} w^{3/2} \right] \tag{1}$$

where w is a complex quantity, and s and g are numerical, real constants. The functions h_1 and h_2 are described and tabulated in the book, "Tables of the Modified Hankel Functions of Order One-third and of their Derivatives," published by the Harvard University Press, Cambridge, Massachusetts, in 1945.

The values of w are uniformly separated, and approximate values are known or can be obtained by extrapolation from those already computed. Starting with an approximate value of w, an accurate value in the range so far investigated can be obtained by two applications of a computing procedure.

For each accurate value of w there is to be found the value of the following product of factors:

$$\begin{gathered}
\pi^{3/2} X^{1/2}\, 12^{-1/3} \left(\frac{1 - s^3}{16 s^2 w^2}\right) \\
h_1(w - sg + sZ_1)\; h_1(w - sg + sZ_2) \\
\left\{ 1 - \frac{h_2(w - sg + sZ_1)}{h_1(w - sg + sZ_1)} \cdot \frac{h_1(w - sg)}{h_2(w - sg)} \right\} \\
\left\{ 1 - \frac{h_2(w - sg + sZ_2)}{h_1(w - sg + sZ_2)} \cdot \frac{h_1(w - sg)}{h_2(w - sg)} \right\} \\
\operatorname{expon}\left[i X s^2 w - i \frac{4}{3} w^{3/2} + i \frac{4\pi}{3} \right]
\end{gathered} \tag{2}$$

1. For the derivation of (1) and (2) see Lincoln Laboratory Technical Report No. 38 (T. J. Carroll, R. M. Ring, February 12, 1954).

where X is the distance of transmission multiplied by a known constant factor, and Z_1 and Z_2 are the heights of the transmitting and receiving antennas, respectively, multiplied by a known constant factor.

It is possible to find all the values of w for which Expression (2) has an appreciable size. The corresponding values of (2) are added together as complex quantities, the sum being the signal strength. A curve of signal strength as a function of X, or distance, may be plotted, and, if desired, a curve of signal strength as a function of Z_2, or height of receiving antenna, may be plotted.

Numerical computation covering about nine-tenths of the work of calculating Expression (2) has been performed on the Whirlwind digital computer and checked by a calculation using a desk machine. A program tape for calculating Expression (2) has been prepared and is being tested.

Measurements of signal strength from the literature will be compared with the computed values.

This work is being carried out by Professor H. B. Dwight of the Lincoln Laboratory at MIT.

194 AN AUGMENTED PLANE WAVE METHOD AS APPLIED TO SODIUM

In selecting a set of approximate wave functions of a periodic crystal potential to form the basis functions of the space in which the energy is to be diagonalized, one must first decide from which bands these functions are to be chosen and secondly from which set of neighbors surrounding the central cell in reciprocal space they are to be chosen. A customary and physically motivated method of choosing the latter is this:

The cells of reciprocal space are divided into rings of increasingly distant neighbors of the central cell, all cells of a ring being equidistant from the origin. Each enlargement of the space in which the energy is to be diagonalized which does not increase the number of bands represented requires adding an entire ring of neighbors.

This space can first of all be divided into invariant subspaces, each subspace being characterized by a particular value of wave vector. Each of these spaces cuts across rings and is characterized by the wave vector of the functions which form the intersection of the subspace with the central cell. This wave vector is referred to as the reduced wave vector of the set of functions which compose the subspace. For certain wave vectors the subspace can in turn be reduced into invariant subspaces each of which is characterized by a ring and by the manner in which it transforms under the operations which leave the subspace invariant. This set of operations commutes with the energy, of course, and is called the group of the wave vector of the space. It can now be arranged that in each subspace of the same symmetry, but of different rings, the basis functions be chosen so that any operation

the earth's surface, at the air-vacuum interface at height g, and at heights above the air. The real parts of the eigenvalues determine the phase velocities of the allowed modes, and the imaginary parts give the attenuation rate with horizontal distance around the earth. The earth radius and wavelengths considered, of the order of a few meters or centimeters, enter also into the numerical values of s and g, the index of refraction gradient and air-layer thickness, respectively. The first step, finding the eigenvalues w of the propagation modes consistent with the boundary conditions, leads to the problem of finding the complex roots of the following equation:[1]

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of the group of the wave vector has the same representation in each space. When the energy is diagonalized only wave functions with the same reduced wave vector and which transform equivalently will combine to form the new basis functions of the space. Thus if these reductions can be effected, a much smaller secular problem need be solved than if one disregarded the invariance of the subspaces and were content to let the secular problem do the job. To effect this reduction the proper basis functions must be set up in each successive ring which is added to the problem.

Mr. M. M. Saffren of the MIT Solid State and Molecular Theory Group is testing a program which will first determine the intersection of the space characterized by a wave vector with the various rings and then will encode the proper coefficients with which to combine functions in the intersection to obtain a subspace of the proper symmetry. The program is restricted to handle crystals only of the cubic group (this includes face- and body-centered crystals). The program is to be first added to Dr. Howarth's program for cubic crystals and will completely mechanize that program, the input becoming extremely simple. The routines of the program being tested will also be used in a program which is to calculate the energy bands in sodium according to the method of symmetrized augmented plane waves.[1]

195 INTESTINAL MOTILITY

The analysis of intestinal-motility records being carried out for Dr. J. T. Farrar of the Gastroenterological Section of the Evans Memorial Hospital involves the autocorrelation and transformation of 80 sets of data. This work has been described under Problem 107 in Summary Reports No. 37 and 38.

Twenty-four sets of data have been autocorrelated using a total time of 46 minutes with 15,900 points input, 2424 points output (101 points output per run). The average run has 700 points of data, giving an average of approximately 2 minutes per run.

These statistics indicate that, because of the high speed and efficiency of WWI, autocorrelations obtained by these methods are not only more accurate but also compare favorably in cost with autocorrelations obtained from special-purpose analog computers. Additional economies in obtaining power spectra will be realized, since the punched-tape output of the autocorrelation program serves as input to the Fourier transform program without further processing.

1. Saffren, M. M.; Quarterly Progress Report, No. 13; July 15, 1954; Solid State and Molecular Theory Group, MIT; pp. 44-48.

196 SINGLE-ADDRESS COMPUTER

The single-address computer (SAC) was developed for use in the MIT 1954 summer session course 6.531. SAC is a modification of the SS computer, which was completed earlier under Problem 140. A detailed description of the SS computer was presented in Summary Report No. 35, Section 5.4. The features which distinguish SAC from the SS computer are the following:

Read-In and Conversion

A SAC program tape, punched by Flexowriter equipment, is read in and converted to WWI binary information by the SAC conversion routine. A new feature which has been added is the option of erasing SAC storage when reading in. Erasure is effected by pressing the lower activate (erase) button before pressing the read-in button. This causes SAC quick-access storage and the simulated magnetic tapes (see below) to be cleared before the program tape is converted.[1] Failure to press the erase button before reading in causes the program information to be superimposed on the previous contents of SAC storage without disturbing the information on the magnetic tape.

The SAC Magnetic Tapes

An auxiliary-storage medium, consisting of four magnetic-tape units, has been incorporated in SAC. Each of the units has a capacity of 990 SAC words. Words are always transferred between SAC storage and magnetic tape in blocks of ten. Thus, each tape is divided into 99 blocks, numbered from 1 to 99.

Three instructions are used in coding for the magnetic-tape units. These provide tape-search, tape-read, and tape-write facilities. The tape-search instruction (mts xyz) is used to select one of the four tape units and to position it before the desired block; the tape-read (mtr alb) and tape-write (mtw alb) instructions operate on the next block in sequence on the most recently selected tape unit.

The magnetic-tape information is stored in WWI not on the magnetic-tape units but on the buffer drum. This leads to faster SAC operation and facilitates the WWI coding; the external behavior of SAC simulates in all respects the use of true magnetic-tape equipment.

1. Provision has been made for automatically resetting the SAC tape units to contain predetermined initial data, whenever this is desired, instead of being erased.

Modification of Instructions

The scope of the arithmetic operations in SAC has been broadened to facilitate the modification of instructions during the course of a program. The cycle-count lines greatly reduce the need for changing instructions in storage, but in certain cases the ability to make such changes is desirable. In the SS computer, operations combining instructions and integers are not permitted. In SAC, such operations may be executed, provided that the result is a proper SAC word. In particular, an integer may be added to or subtracted from an instruction to produce an instruction with a modified address, and two instructions with the same operation section may be subtracted to produce an integer equal to the difference of the address sections.[1] These procedures are similar to operations normally executed by many digital computers (including WWI) and add considerable flexibility to the treatment of instructions in SAC.

The Instruction rip0

The SAC instruction code contains one additional instruction, rip0, not found in the SS code. This instruction permits the programmer to call for the read-in of a new program tape. When rip0 is executed, control is transferred to the SAC conversion routine. The instruction is equivalent, in effect, to pressing the read-in button without pressing the erase button. The tape read in using the rip0 instruction must therefore be complete within itself; no reference may be made to symbolic addresses which were assigned in preceding program tapes unless these addresses are reassigned in the new tape.

The Permanent Table of Constants

There is available in SAC a set of fixed quick-access registers whose contents may be read but not changed by the routine. Register 0 contains the integer 0, and registers 290 to 298 contain in sequence the integral powers to ten from $10^0 = 1$ to $10^8 = 100,000,000$. These constants may be utilized by the programmer in exactly the same manner as integers stored within his routine, except that any attempt to modify them will result in a post mortem.

The Treatment of "Excess"

Arithmetic operations on integers in SAC may yield results which are too large to be stored in a SAC memory location. To allow for the generation of such integers, the SAC

1. In all cases, the counter letter, if any, is ignored.

accumulator is made double the length of a memory location. The part of the accumulator which contains any overflow that may be generated is called the "excess." Any attempt to transfer the contents of the accumulator to storage when the contents of excess are not zero will result in a post mortem.

In the SS computer, the instruction txi alc causes the contents of excess to be transferred to register alc and excess to be cleared. The contents of excess, however, are not simply related in the decimal system to the integer which originally was generated.[1] Consequently, the use of the txi instruction is in most cases quite awkward, and it is frequently more convenient to clear excess using an arithmetic division by a suitable power of ten. This is analogous to the decimal-shift-right found in many decimal digital computers. The txi instruction has been abandoned in SAC in favor of the more easily understood and more logical arithmetic division. The jix alc instruction, used to determine whether excess is occupied, has been retained.

Timing of SAC Programs

At the termination of a SAC program, either by the instruction Stp0 or by a computation post mortem, a fictitious time is automatically printed by the computer, indicating how long a physically existent SAC would have required to execute the program performed. This time is computed during the course of the program, giving a preassigned weight to each instruction as it is carried out. The weights assigned to the instructions are based on 1 millisecond for most operations, with proportionally greater times required for multiplication, division, and the operations using input and output equipment.

In calculating the time, it is assumed that the magnetic-tape-search (mts) operation (but not tape read and write) may proceed in parallel with other computer operations until either the search is completed or another tape instruction is given. The time for multiplication and division, when the address section of these instructions refers to the permanent table of constants,[2] is made proportional to the power of ten involved and in all cases is less than the time required to perform the same operation using an ordinary SAC location. This is done because such multiplication or division would ordinarily be accomplished using a numerical-shift operation, which in many computers requires less time than straightforward multiplication and division. In all other cases, the time is calculated assuming that no operations may proceed simultaneously and that the time required is independent of the numerical quantities and memory registers employed.

1. Actually, this integer is: (contents of excess) $\times 2^{27}$ + (contents of accumulator).
2. See above.

Automatic Post Mortems

The SAC conversion and computation post mortems are virtually identical to those of the SS computer. The computation post mortem in SAC indicates the positions of the four tape units and specifies the most recently selected unit, if the tape has been used. No other changes in the post mortem have been made necessary by the modification of the computer instructions.

197 THREE-ADDRESS COMPUTER

A hypothetical three-address computer (called TAC) was developed during the past quarter by staff members of the S&EC Group. TAC was used during part of the 1954 summer-session course 6.531 (Digital Computers: Business Applications). A description of TAC is given in Section 5.2 of this report.

198 STUDENT PROBLEMS CODED FOR SAC AND TAC

This problem number was set up for the use of students enrolled in the special summer-session course 6.531 (Digital Computers: Business Applications).

All of the students in the course coded and debugged a problem assigned for solution on the simulated three-address computer called TAC (developed under Problem 197). The problem consisted of determining and printing out the day of the week corresponding to an arbitrarily specified date.

Most of the students also worked on a second problem to be solved on the simulated single-address computer, SAC (see Problem 196). For this second problem, the students were permitted to select from among three suggested problems covering the maintenance of bank-balance records, payroll calculations, and an autocorrelation calculation to detect a seasonal sale variation.

199 LAMINAR BOUNDARY LAYER OF A STEADY, COMPRESSIBLE FLOW IN THE ENTRANCE REGION OF A TUBE

In connection with research on heat-transfer coefficients, recovery factors, and fraction coefficients for supersonic flow of air in a tube, a theoretical investigation of the characteristics of the laminar boundary layer in the entrance region has been carried out by Dr. T. Y. Toong of the MIT Mechanical Engineering Department. Partial differential equations of continuity, momentum, and energy were developed for the boundary layer. These were then transformed into a series of ordinary differential equations to be solved by WWI for several entrance Mach numbers and thermal conditions at the tube wall.

First, solutions are to be obtained for the case of constant viscosity and thermal conductivity. Then, the effects of temperature dependence of these properties are to be studied.

Solutions of the first set of differential equations for the case of constant viscosity and thermal conductivity have been obtained for six cases of different entrance Mach numbers and thermal conditions at the tube wall. This represents about 6% of the entire job to be done by WWI.

Work is being done to solve the second set of differential equations corresponding to the same six cases.

The coding has been carried out in the "algebraic system" developed by Dr. J. H. Laning under S&EC Problem 108. This system makes use of Gill's modified fourth-order Runge-Kutta method.

200 A STUDY OF RECURRENT EVENTS

A study of recurrent events in sequences of binary numbers is being made by Miss Bergitte Jensen and Dr. G. P. Dinneen of Lincoln Laboratory. In each experiment two Bernouilli sequences are generated using a pseudorandom number generator. The basic sequence has a probability, $p = P_n$, of a one which is small, less than 0.1. This is denoted as a noise sequence. The signal sequence has a probability, $p = P_s$, of a one which varies from about 0.2 to 0.9. The signal sequence is generated for a time which is short compared to the length of the noise sequence.

Various sampling techniques are being tried to determine the optimum statistical test for determining the presence of the signal sequence and its position. The more common statistical problem is that of deciding whether a finite sample comes from a noise region or a signal region. Here we observe a finite sample, part of which is from noise and part from signal, and must determine the point of transition. Two methods are being tried. One is the success-run technique, where a particular sequence such as 0 0 0 0 0 is searched for. The other is the density method, where the density over some interval is measured as the interval moves through the sequence. Whenever the density exceeds a certain threshold, we say the sample is from a signal region.

Programs for both of these methods have been coded and preliminary results obtained. It is expected that this study may be completed during the next quarter.

201 STUDY OF THE AMMONIA MOLECULE

A self-consistent calculation of the electronic wave functions and binding energy of the ammonia molecule has been carried out. The method, which has been described in detail in previous Summary Reports under Problem 144, is this: each 1-electron molecular orbital is expressed as a linear combination of the set of atomic orbitals belonging to the nitrogen atoms and to the three hydrogen atoms. The total molecular wave function is the determinant with the 1-electron molecular orbitals as elements. The expectation value of the Hamiltonian is expressible as a quartic form in the coefficients of the linear combinations. Insistence that the energy be stationary with respect to variation of these coefficients demands the diagonalization of an effective Hamiltonian matrix which itself contains the unknown coefficients. The problem is solved iteratively in what is termed a self-consistent manner. A first guess is taken for the coefficients, a new set is determined by the diagonalization of the effective Hamiltonian, this new set is fed back in, and so on until the input and output of a cycle agree to sufficient accuracy.

The matrix components of the effective Hamiltonian are composed of kinetic and potential energy integrals over the fixed set of 1-electron functions. Actually, these were taken to be not the primitive atomic set but orthogonalized and symmetrized combinations of them. The integrals over the atomic functions were evaluated on desk computers and IBM equipment. The integrals over the symmetrized combinations were obtained by means of a congruent transformation applied to a matrix whose elements were the atomic integrals. This was done on Whirlwind (a 36 x 36 congruent transformation).

With these integrals, the self-consistent calculation was done on Whirlwind. Various criterions of self-consistency were tried so as to judge the stationary property of the energy. The results are discussed by Dr. H. Kaplan in the 15 September Progress Report of the Solid State and Molecular Theory Group at MIT.

202 CALCULATION OF VERTICAL ANTENNA COVERAGE SKELETON

For several months investigations on vertical coverage interference patterns have been carried on by A. F. Bartholomay of Lincoln Laboratory. Complete detailed discussion on this subject can be found in his forthcoming report entitled "On the Calculations of the Vertical Coverage Interference Pattern."

In brief, however, an interference pattern can be analyzed into two components: (1) the skeleton structure which consists of axes through the center of all lobes and loci of all null points for constant point differences, and (2) either a family of envelopes of corresponding lobe points (the general case); or (in the flat-earth case) sinusoidal segments approximating the actual lobe shapes. The difficulty in obtaining one complete pattern, notwithstanding several patterns which actually are needed, is twofold: (1) numerous formulas are required to determine a single point; and (2) the number of lobes in such a pattern is directly proportional to the height of the antenna (h_1) above the surface. For example, for an antenna height of 10,000 ft and a 100-mc radar, there will be roughly 2000 lobes. If we allow 12 points as sufficient to describe each lobe, we would need 24,000 separate calculations for the complete diagram.

Because of the length of the computations, programming the problem for Whirlwind I seemed the best approach. Since the entire problem is time consuming even for a large-scale digital computer, we considered only obtaining the skeleton structure of the pattern. We did develop a successful program in the (24, 6) system where, for a given antenna height of 10,000 ft, a skeleton pattern of the first 30 lobe lines was obtained. More explicitly, the program printed out in decimal form the X_o and Y_o coordinates for each n lobe line ($1 \leq n \leq 30$) and for the curve of the earth's surface; and, at the same time, using the automatic curve-plotting routines, the program gave a scope display of the entire skeleton structure on one frame. The actual computer time consumed less than 10 minutes; equivalent calculations performed manually took several weeks. However, unforeseen complications in the original method of computation made us question the continuation of the problem on Whirlwind I. For the program to operate, limits must be set on d_1, which is a function of n but which varies greatly for different n. Only through performing manually the entire calculation can such limits be found. Therefore, because of this and other inherent difficulties in the problem itself (see above report), we conclude that, for the present, this problem is best handled by desk calculations.

204 EXCHANGE INTERVALS BETWEEN REAL SLATER ORBITALS

It is reasonable to expect that our ideas of chemical behavior can be sharpened when we are able to compute this behavior from fundamental theory. A theory that ascribes coulomb forces, spins, and correct statistical properties to atomic nuclei and electrons with quantum mechanics in the form of Schroedinger's equation is believed to be sufficient to explain all important chemical phenomena. Cases have been computed (Hylleras - the helium atom; James and Cooleridge - the hydrogen molecule) to demonstrate the remarkable agreement of deduced behavior with experimental phenomena. Nevertheless, to date no ambitious computations closely related to the fundamental theoretical basis have been essayed.

With simplifying assumptions, it is felt that such computations can be made feasible if the distribution (or wave functions) of the electrons are expanded in a quickly convergent set of functions which form a complete set and, as orbitals of a single atom, have in a single function almost uniquely the properties of an orbital electron. Slater orbitals are such functions.

An initial and computationally most difficult step in such a procedure is to compute integrals where the integrand is the product of two or four such orbitals and an operator. It is this step that gives impetus to the "Integrals Project" now being carried on at the University of Chicago under professors Mulliken and Roothaan. It is the eventual aim of the project to have all integrals necessary to molecular computations readily accessible.

The specific integral to be computed on Whirlwind I is a two-center exchange integral, which is most easily explained in terms of the equivalent electrostatic problem.

This problem involves the computation of the electrostatic interaction of two charge densities, $\Omega(1)$, $\Omega(2)$, by Coulomb's law.

$$\int \frac{\Omega(1)\Omega(2)}{r_{12}} dV_1 dV_2$$

where the volume integrations are over all space. Each charge distribution is the product of two functions of the form

$$\psi = (2\xi)^{n+\frac{1}{2}} / [\pi(2n)!]^{\frac{1}{2}} r^{n-1} e^{-\xi r} \left[\frac{(2\ell+1)(\ell-m)!}{2(\ell+m)!}\right] P_\ell^m(\cos\phi)\, f^m(\phi)$$

$$f^m(\phi) = \begin{cases} 1/\sqrt{2} & m = 0 \\ \left.\begin{matrix}\cos m\phi \\ \sin m\phi\end{matrix}\right\} & m \neq 0 \end{cases}$$

where the parameter ranges are $\ell + 1 \leqq n$, $n \leqq \ell$, and $\xi \geqq 0$. The two functions whose product makes a charge distribution are located about distinctly separate origins.

A transformation of the variables to elliptic coordinates is effected and $\frac{1}{r_{12}}$ expanded by the Neumann expansion.

The resulting integral can be separated allowing the construction of an infinite series of integrals where the quadrature has been reduced from a sixfold to a double quadrature, the limit of the first quadrature being the argument of the second.

$$I = h \sum_{\ell = M}^{\alpha} I_\ell^M$$

$$I_\ell = K \int_1^{\alpha} d\xi (\xi^2 - 1)^{-1} f^M(\xi, \alpha)\, f'^M(\xi, \bar{\alpha})$$

α is a function of the ξ's arising from $\Omega(1)$ and R. $\bar{\alpha}$ is similarly defined for $\Omega(2)$.

$$f^M(\xi, \alpha) = \frac{1}{P_\ell^M(\xi)} \int_1^{\xi} dx\, P_\ell^M(x)(x^2 - 1)^{M/2} e^{-\alpha x} w_\ell^M(B, x)$$

where $w_\ell^M(B, x)$ is a polynomial in integral powers of x, the coefficients of which are determined as functions of all the parameters arising in a given charge distribution.

The coefficients arising in the $w_\ell^M(B, x)$ polynomials are constructed from a set of spherical Bessel functions of imaginary argument and 1/2-integer order. Two arguments

will be necessary for each total integration, and the process is to be carried out only once during the computation of a total integral. These functions are derived from the initial construction of four functions by means of a recursion formula. From these Bessel functions further functions will be constructed with the construction of each I_ℓ by means of a simple recursion. At the outset it will be necessary to construct a table of coefficients. This table is representative of the multiplication of the two ψ functions together. The major portion of the computation is the numerical quadrature necessary to obtain an I_ℓ. The integration from 1 to ∞ is converted to an integration from 1 to 0 to give a finite range to the integration. The procedure will be to evaluate the integrand (including the inner integrations) exactly and approximate the outer integration by some "numerical quadrature" procedure.

To date, the initial coefficient table -- to be used in constructing the $w_\ell^M(B, x)$ polynomial -- has been coded by P. Merryman and is now being tested.

In view of the fact that the heaviest portion of the computational labor is in the numerical integration, rapidly converging numerical-quadrature schemes have been investigated. Of these schemes the most powerful, Gaussian, has proved adaptable, mutatis mutandis, and gives promise of decreasing the total necessary computations by the order of 70% of those necessary when using Simpson's rule. This scheme has been flow diagrammed for the specific case and will shortly be coded and tested.

A subroutine giving spherical Bessel functions of imaginary argument and 1/2-integer order, as well as square roots and exponentials, in (24, 6) has been completed and tested by F. J. Corbató of the MIT Physics Department and is now ready for incorporation into this scheme.

The procedure for computing an exchange integral is now sketched in sufficient detail. The remaining effort will be almost entirely devoted to the tasks of coding, testing, and assembly.

Reference: Ruedenberg, Klaus, "The Two-Center Exchange Integrals" (Part II of "A Study of Two-Center Integrals Useful in Calculations on Molecular Structure"), Journal of Chemical Physics, Vol. 19, No. 12, pp 1459-1477, December 1951.

205 ELECTRON LATTICE INTERACTION IN SOLIDS

Because of the large dipole moments associated with the long wavelength optical modes of vibration in ionic crystals, the coupling between these modes of vibration and the motion of a conduction-band electron is strong, too strong to be treated by perturbation theory. It was even conjectured at one time that such an electron could be trapped in the potential well created by the polarization charge it itself had induced. Although no experimental evidence for such a self-trapped electron has been found, it is still expected that the effective mass of a conduction electron together with its associated lattice polarization could be very large.

Various approaches to the problem of a single electron interacting strongly with the optical modes have been proposed. Fröhlich, Pelzer, and Zienau[1] and Gross[2] have worked in a Fock space for the phonon assembly introducing cutoffs above 1, 2, or 3 quanta, then attempting an exact solution of the approximate problem remaining. Lee, Low, and Pines[3] and Gurari[4] have applied intermediate coupling theory in what amounts to a Hartree-Fock approach in the momentum representation of Fock space. Various Russian writers, including Bogolyubov,[5] Tyablikov,[6] and Pekar,[7] have worked in an electron-lattice configuration space making various approaches to an adiabatic approximation.

Basic to all these approaches are certain assumptions. The periodic potential of the lattice is taken into account through the introduction of an effective mass. The lattice is treated as a harmonically vibrating continuum of polarization charge in a phenomenological Hamiltonian introduced by Fröhlich.[8] The analyses are restricted to small momenta, i.e., slow electrons.

Several questions have been raised: Haken[9] in treating a highly specialized model has suggested that the effect of the periodic potential may be considerable. Lee and Pines claim to obtain an exact solution in the limit of strong coupling, yet Pekar, using a variational method, seems to obtain lower energies for sufficiently strong coupling. Lastly, the Fock-space methods suggest an effective mass increasing sharply with momentum, but these methods are valid only for small momenta.

In order to resolve some of these difficulties it is desired to construct a model which can be solved with sufficient exactness to serve as a yardstick in determining the validity of the various approximations. A possible model has been investigated exactly by Gross,[10] who determined the solutions to the eigenvalue problem arising when an otherwise free electron is allowed to interact with only one traveling lattice wave and all motion is confined to one dimension. A possible limitation of this model is its spatial asymmetry. For sufficiently strong coupling, for example, Gross finds the state with minimum energy has nonzero momentum. Conceivably this is a fault of the model's asymmetry. To determine whether or not the asymmetry is truly a limitation to Gross's model, the problem of a free electron interacting with two degenerate modes (right- and left-going) in one dimension has been set up by T. D. Schulz of the MIT Solid State and Molecular Theory Group. The matrices of order $\frac{(n+1)(n+2)}{2}$ arising on neglect of all states having more than n phonons excited are being diagonalized for divers values of coupling, oscillator wavelengths, and total

* Numbered superscripts refer to references at the end of this problem.

momentum (using a routine developed by Dr. Meckler under Problem 134) with the following aims:

1. To see how the effect of the interaction on the ground state depends on wavelength;
2. To see if the asymmetry of Gross's model is important;
3. To provide an accurate yardstick for evaluating adiabatic methods used on real crystals in the event the Gross's model is invalidated; and
4. To provide a starting point for considering the effects of a periodic potential and a discreteness in the vibrating lattice.

Two values of the coupling constant (intermediate and strong) and two wavelengths (intermediate and long) are being investigated. For intermediate coupling good convergence has been obtained. For strong coupling it is found that a better criterion than total number of quanta must be used in cutting off the infinite secular equation to obtain good convergence. Dr. Meckler is considering modifications in the present routine which would include only those states with n lowest diagonal matrix components.

References:

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2. Gross, E., Tech. Report 55, Lab. for Insulation Res., MIT (1952).
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10. Gross, E., Phys. Rev. 84, 818 (1953).

206 ELECTRON ENERGIES OF THE MOLECULE H_2^-

The complete problem, undertaken by A. Dalgarno of Queen's University of Northern Ireland in association with the MIT Physics Department, was the evaluation of the rates of the collision processes

$$H + H^- \rightarrow H^- + H \qquad \text{(1) charge transfer}$$

$$H + H^- \rightarrow H_2 + e \qquad \text{(2) associative detachment}$$

and, in addition, (3) the evaluation of the mobility of H^- in a gas of atomic hydrogen. Such processes are of importance in the physics of the upper atmosphere and of the sun.

The basic data required for (1), (2), and (3) are the electronic energies of the two lowest states of the molecule H_2^-. (It may be mentioned incidentally that the calculations will also provide useful information on the dissociation energy and electron affinity of H_2^-.) The energies of H_2^- can be expressed in terms of certain molecular integrals, and it is for the evaluation of these that the Whirlwind I computer has been used.

The integrals involved have the forms:

$$\int r_a^n e^{-\alpha r_a} r_b^n e^{-\beta r_b} dr \;, \quad \int r_a^n e^{-\alpha r_a} \nabla^2 r_b^n e^{-\beta r_b} dr \;,$$

$$\int r_{1a}^n e^{-\alpha r_{1a}} r_{1b}^n e^{-\beta r_{1b}} r_{2a}^k e^{-\gamma r_{2a}} r_{2b}^\ell e^{-\delta r_{2b}} (1/r_{12})\, dr_1\, dr_2$$

where r_{1a} (or r_a), etc., is the distance of electron 1 from nucleus A, r_{12} is the distance of electron 1 from electron 2, α, β, γ, δ are parameters, and m, n, k, ℓ are integers which may be negative. Depending on the values of the various parameters, essentially six different integrals arise, known as

1. Overlap
2. Coulomb attraction
3. Kinetic energy
4. Coulomb repulsion
5. Hybrid
6. Exchange

and they are required for a large number of values of α, β, γ, δ and the nuclear separation R of which the integrals are a function.

Integrals 1, 2, and 3 were already available in a program written by F. J. Corbató of the MIT Physics Department. It was hoped to program 4, 5, and 6, but because of time limitations only 4 and 5 were completed. However, 6 is being programmed under Problem 204 by P. Merryman.

The integrals which were programmed and for which results were obtained are specifically

$$\int e^{-\alpha r_{1a}} e^{-\beta r_{2b}} (1/r_{12})\, dr_1\, dr_2 \quad \times \quad \begin{cases} 1 \\ r_{1a} \\ r_{1a} r_{2b} \\ r_{1a}^2 \end{cases}$$

$$\int e^{-\alpha r_{1a}} e^{-\beta r_{2b}} e^{-\gamma r_{1b}} (1/r_{12})\, dr_1\, dr_2 \quad \times \quad \begin{cases} 1 \\ r_{1a} \\ r_{1b} \end{cases}$$

By an elementary expansion of $1/r_{12}$ in terms of spherical harmonics, it may easily be shown that all these integrals are expressible as somewhat lengthy combinations of products of auxiliary integrals

$$A_n(\lambda) = \int_1^\infty e^{-\lambda x} x^n dx \;, \quad B_q(\mu) = \int_{-1}^{+1} e^{-\mu x} x^q dx$$

A routine for generating these functions was available.

208 INTERCEPTOR FLIGHT-CONTROL PROBLEM

A system of 14 first-order nonlinear differential equations representing an interceptor flight-control problem was solved by M. Merwin of the MIT Dynamic Analysis and Control Laboratory using the Gill adaptation of the fourth-order Runge-Kutta method. The Whirlwind solution showed that the equations were unstable. These results, reproduced later on an analog computer, proved to be most useful, since the system defined by the differential equations was thought to be an optimum system.

209 NUMERICAL SOLUTION OF HOMOGENEOUS LINEAR DIFFERENTIAL EQUATIONS IN THE QUADRATIC POLYNOMIAL COEFFICIENTS

This problem was selected by Dr. J. C. P. Miller of Cambridge University, Cambridge, England, as one that not only would provide a generally useful and interesting routine but could be completed during his brief visit at the Digital Computer Laboratory.

The equation to be solved is

$$(a_1x^2 + a_2x + a_3)\frac{d^2y}{dx^2} + (b_1x^2 + b_2x + b_3)\frac{dy}{dx} + (c_1x^2 + c_2x + c_3)\,y = 0,$$

starting at $x = x_0$, with $y = y_0$, $\frac{dy}{dx} = y_0'$, at interval h in x.

The method of solution is to obtain a recurrence relation for terms in the Taylor expansion at each x and to find values of y and y' at x + h, by summation, and then start again with the new value of x.

Writing the equation as

$$p(x)\,y_2 + q(x)\,y_1 + r(x)\,y = 0$$

and differentiating n times we get

$$py_{n+2} + (np' + q)\,y_{n+1} + \left(\frac{n(n-1)}{2}p'' + nq' + r\right)y_n + \left(\frac{n(n-1)}{2}q'' + nr'\right)y_{n-1}$$
$$+ \frac{n(n-1)}{2}r''y_{n-2} = 0.$$

Then if we write $T_n = \frac{h^n y_n}{n!}$ or $y_n = n!\,T_n h^{-n}$, we obtain

$$(n+1)(n+2)pT_{n+2} + (n+1)(np' + q)hT_{n+1} + \left(\frac{n(n-1)}{2}p'' + nq' + r\right)h^2T_n$$

$$+ \left(\frac{n-1}{2}q'' + r'\right)h^3T_{n-1} + 1/2\,r''h^4T_{n-2} = 0.$$

This defines T_{n+2} for n = 0, 1, 2, ... if we take $T_{-2} = T_{-1} = 0$, $T_0 = y$, and $T_1 = h_1$. Then

$$y(x+h) = T_0(x+h) = T_0 + T_1 + T_2 + \ldots$$

$$hy'(x+h) = T_1(x+h) = T_1 + 2T_2 + 3T_3 + \ldots$$

gives the new T_0 and T_1. The processes cease when two successive terms are negligible.

This equation covers many familiar and unfamiliar functions; it has been used with

e^{x^2}	satisfying $y'' - 2xy' - 2y = 0$
$\cos x$	satisfying $y'' + y = 0$
$\int_0^x e^{x^2}dx$	satisfying $y'' - 2xy' = 0$
x^{10}	satisfying $xy'' - 9y' = 0$
$y = \log x$	satisfying $xy'' + y' = 0$

showing that nearly seven figures can be readily obtained. The greatest error occurred with e^{x^2}, where units in the seventh figure accumulated in eight steps, corresponding to a factor exceeding 10^6 in the increase of the function.

211 SERVO RESPONSE TO A COSINE PULSE

This problem is concerned with extending the curves in Figure 25-16, page 705 of Instrument Engineering, Volume II, by Draper, McKay, and Lees. It is desired to find the maximum response x(t) for 1440 parameter pairs (ζ, T_p/T_n) in the equation

$$(1/w_n^2)\,\ddot{x} + (2\zeta/w_n)\,\dot{x} + x = G(t), \text{ where } T_n = 2\pi/w_n,$$

$$\dot{x} = dx/dt,\ \ddot{x} = d^2x/dt^2,\ \dot{G}(t) = (1/2)\left[1 - \cos(2\pi t/T_p)\right] \text{ for } 0 \le t \le T_p,$$

$$\text{and } G(t) = 0 \text{ for } t > T_p.$$

The parameters ζ and T_p/T_n are from the range $0.01 \le \zeta < 5$, $0.1 \le T_p/T_n \le 10$.

Analytic expressions for x(t) may be found in the above-mentioned book, but these were found impractical for determining the maximum response. Accordingly, a step-by-step approximation to the response curve using Taylor series is employed. The step-by-step procedure is stopped when the derivative becomes negative, and an interpolation is carried out back into the last step. This procedure was applied using parameters different from those of the present problem by Dr. J. H. Laning, Jr., in coding the problem for a card-programmed calculator. Coding of the problem using the CS II system was accomplished by Dr. J. M. Stark of the MIT Instrumentation Laboratory.

3. OPERATION OF WHIRLWIND I

During the past quarterly period the Whirlwind Computer System reliability was high. According to estimates by computer operators, 96% of the 886 hours of assigned operation time was usable. A detailed account of the assignment of computer time to the various major activities is given in Fig. 3-1.

Work by the systems group on the central computer has included modernization of the arithmetic-element control, reorganization of the power-supply control for a large segment of the terminal equipment, and some equipment relocation and circuit changes for easier, less-frequent trouble shooting. Marginal-checking facilities, extended to cover more of the terminal equipment, have already proved their value by revealing some circuit shortcomings now corrected.

Two additional drum groups of auxiliary storage added to the 15 already available to the computer increase drum storage from about 31,000 to about 36,000 registers.

Activity	Hours Per Week													
	July				August				September					Total Hours
	2-8	9-15	16-22	23-29	30-5	6-12	13-19	20-26	27-2	3-9	10-16	17-23	24-30	
Marginal Checking	4	4	4	4	4	4	4	4	4	4	4	4	4	52
Installation	0	5	8	8	10	8	8	9	8	0	7	7	8	86
Maintenance	15	31	22	25	30	36	27	20	22	11	38	36	26	339
Terminal Equipment Testing	28	43	39	35	28	28	30	30	34	30	29	25	33	412
Technician Instruction	5	4	4	4	4	4	4	4	4	4	4	4	4	53
Scientific and Engineering Computation	46	47	50	50	49	41	53	60	50	28	33	43	43	593
Other Applications	25	14	21	21	24	21	26	22	27	19	19	23	31	293
Total Hours	123	148	148	147	149	142	152	149	149	96	134	142	149	1828

Fig. 3-1. Allocation of Computer Time

3.1 Systems Engineering

Most of the circuitry and logic of the arithmetic element of the computer is contemporary with the original system installation. The elementary state of the art at that time led the original designers to build into their circuits a large degree of flexibility against unforeseen circumstances. Experience in computer operation has shown some of the original equipment to be unnecessarily complicated; where practicable WWI is being improved in this respect.

Revisions of the arithmetic-element control included standardization of circuitry and changes in logic. These modifications to the stop counter and divide control eliminated excess circuitry, reduced the time required for the divide operation by one-third, and increased the operating margins of the system.

A large part of the terminal equipment for WWI, one floor below the main section of the computer, was installed at a later date than the bulk of the WWI circuitry, and a separate power-supply-control system was designed for it. It has long been evident that the power-supply-control system left much to be desired because of the inaccessibility of the circuitry and because the power is not turned off in proper sequence in all modes of shutdown. This results in writing between the slots on the magnetic drums and has materially reduced drum reliability. In addition to these shortcomings, it is now felt that the present independent control of the auxiliary drum and buffer drum is not necessary.

A new power-supply-control system eliminates these objections mentioned and reduces the number of independent controls for the two groups of equipment.

The magnetic-tape equipment in use with WWI can be logically divided into two separate systems, the auxiliary memory (which has the exclusive use of two tape units) and the delayed-printout system (which shares the other three with the computer). The magnetic-tape system, in its auxiliary-memory capacity, has been operating satisfactorily with the computer during the past quarterly period. The delayed-printout equipment has been giving some trouble, and several changes were made in the system to improve its reliability. Circuit modifications improved the operating margins, and a relay in the control register allowed the Flexowriters more time to operate. Additional relays have been installed to reduce unwanted signal coupling. The equipment is being moved to a new position where the panels will be more accessible for servicing.

3.2 Terminal Equipment

In the past two years the input-output system for WWI has grown so rapidly that marginal-checking facilities were sometimes not added as new equipment went into service. When this omission was made good, marginal-checking lines frequently were not optimized in their arrangement. During this quarter new marginal-checking circuits were installed, and old lines for the in-out system were reassigned to bring them up to the standards of the rest of the computer.

Some developmental work was necessary prior to this installation because of the nature of several of the circuits associated with the terminal equipment. A method of marginal-checking the crystal gates in the "and" and "or" circuits for the intervention registers was developed so that it is now possible to pinpoint weak crystals in these registers.

When marginal checking was extended to the gas-tube indicator-light register, hitherto unsuspected weaknesses of the circuit were revealed by the low and rapidly deteriorating margins. As a result of this discovery, the extinguishing circuit was redesigned to make the tube requirements less critical and to give more reliable operation.

Auxiliary-storage capacity of the buffer drum has been increased from about 8000 to about 12,000 registers, each of 17 bits, by using Groups 2 and 3 for this purpose.

The source of intermittent crosstalk between the buffer- and auxiliary-storage sections of the buffer drum has been located and removed. Occasional buffer-drum parity alarms over a period of several weeks are attributed to this fault. The change should improve the reliability of the buffer drum to the point where it approaches that of the auxiliary drum.

4. CIRCUITS AND COMPONENTS

4.1 Vacuum Tubes

4.1.1 Vacuum-Tube Life

During the third quarter of 1954 the WWI computer operated for 1680 hours.

Vacuum-tube life has been calculated for five different tube types as outlined in Summary Report No. 36. A summary of this information together with previous data is shown below.

			FAILURE RATE, PER CENT PER 1000 HOURS		
Tube Type	1952	1953	First Quarter 1954	Second Quarter 1954	Third Quarter 1954
7AD7/SR1407/6145	2.00	3.3	1.75	1.6	1.4
7AK7	0.26	0.43	0.5	0.6	0.3
6080/6080WA/6AS7G		6.6*	1.1	1.5	1.4
5965		0.2*	0.4	0.3	0.2
6BL7GT		0.7*	0.3	0.5	0.5

*Last quarter 1953 only

There are no major changes in these figures from the second quarter.

Several life tests are still active. That of the GE 5965 has reached 5400 hours with one section conducting and the other cut off. Monotonic shifts in contact potential, which have caused the grids of the cutoff sections to become more positive and those of the conducting sections to become more negative, have occurred. The average difference between the sections is about 0.7 volt. Considerable grid emission has developed on the cutoff sections while the grid currents of the conducting sections are normal. No interface impedance has been observed.

A life test on the RCA A4688B (developmental 5965), being run with one section conducting and the other cut off, has reached 2000 hours. Some grid-cathode leakage has begun to develop, predominantly on the conducting sections. Four of the 15 cutoff sections show interface impedance ranging from 1 to 3 ohms with a fifth section showing 16 ohms. No interface is present on the conducting sections.

The life test of special 5687's with A31 Cathaloy (tungsten-nickel) cathode sleeves

Type	Total in Service	Hours of Operation	Reason for failure; number failed				
			Change in Characteristics	Shorts, Opens,	Breakage	Burn-out	Gassy
6145/7AD7/ SR1407	3866						
6145		0 - 1000	1	2			
		1000 - 2000	1	1			
		2000 - 3000		6			
		3000 - 4000	2	2			
		4000 - 5000	1	1			
		5000 - 6000	1	7			
		6000 - 7000		6			
		7000 - 8000		2			
		8000 - 9000		1			
		9000 - 10000		1			
7AD7		8000 - 9000		1			
		11000 - 12000		2			
		14000 - 15000		1	1		
		22000 - 23000	3				
		24000 - 25000	3		1		
		25000 - 26000	8	7			
		26000 - 27000	15				
		27000 - 28000	7				
SR1407		6000 - 7000	1				
		7000 - 8000	2	1	1		
7AK7	3219	0 - 1000			1		
		1000 - 2000		2			
		2000 - 3000	1				
		3000 - 4000	3	3			
		7000 - 8000	2	1			
		8000 - 9000		2			
715C	124	no hours kept	6		1		
6AU6/6136	314						
6136		0 - 1000		1	5		
		2000 - 3000		3			1
6AU6		0 - 1000	1	2			
		1000 - 2000		1	2		
		2000 - 3000		1	1		
		5000 - 6000		1			
		6000 - 7000	1	1			
		8000 - 9000	3	5			
6BL7	490	3000 - 4000	1				
		6000 - 7000		1			
		7000 - 8000		2			
6L6G/5881	79						
6L6G		5000 - 6000	1				
		10000 - 11000	3				
		11000 - 12000	1				
5881		0 - 1000		2			
		2008 - 3000		1			
		3000 - 4000	1				
6SH7	3	2000 - 3000	1				
6SN7	307	25000 - 26000		1			
		26000 - 27000		3			
		27000 - 28000	1	1			
6X5GT	25	6000 - 7000		1			
		9000 - 10000		1			
6Y6G	322	11000 - 12000		1			
		12000 - 13000		3			
		24000 - 25000	3	1			
		25000 - 26000	2	1	1		

Fig. 4-1. WWI Tube Failures

Type	Total in Service	Hours of Operation	Reason for failure; number failed				
			Change in Characteristics	Shorts, Opens,	Breakage	Burn-out	Gassy
C16J	12	4000 - 5000	1				
OD3	23	2000 - 3000	1				
2D21	204	0 - 1000	1				
		1000 - 2000	3				
		2000 - 3000	1				
		3000 - 4000	4				
		4000 - 5000	1				
		5000 - 6000	1				
		13000 - 14000	1				
3E29/829B	141						
829B		5000 - 6000	1				
3E29		6000 - 7000	1				
		26000 - 27000	1				
5Y3	6	3000 - 4000		1			
6AC7	9	2000 - 3000		1			
6AG7	81	24000 - 25000		1			
6AN5	102	2000 - 3000		1	1		
		3000 - 4000		1			
6AS7G/6080/ 6080WA	718						
6080		1000 - 2000	1	1			
		2000 - 3000		1			
		3000 - 4000	2	9			
6080WA		5000 - 6000	2	1			
6072	30	2000 - 3000	1				
12AU7/5963	450						
12AU7		2000 - 3000	2	1			
5963		0 - 1000		5			1
		1000 - 2000	2	2			
		2000 - 3000	6	6			
		3000 - 4000	4				
		4000 - 5000		1			
		5000 - 6000	2	1			
		7000 - 8000	8				
		8000 - 9000	6	1			
		9000 - 10000	7	2			
		12000 - 13000	2				
5651	23	4000 - 5000			1		
5670	47	13000 - 14000		1			
5687	85	3000 - 4000			1		
		10000 - 11000		2			
		16000 - 17000	1				
5965	786	0 - 1000		1			
		7000 - 8000		1			
5998	34	4000 - 5000	1				
		7000 - 8000		1			

has reached 3400 hours with one section conducting and the other cut off. Considerable grid emission has developed on the cutoff sections; grid currents in the conducting sections are low. The difference in plate current at zero bias between the conducting and cutoff sections of a given tube is quite small. This is a considerable improvement over the standard 5687. Three of the ten conducting sections show interface impedance ranging from 3 to 15 ohms, while no interface is apparent on the cutoff sections.

A second life test involving tungsten-nickel A31 Cathaloy cathode sleeves, being run on some special GE Z-2177's (improved 5965), has reached 2000 hours with one section conducting and the other cut off. In general the grid currents are low indicating that grid emission may not be a problem. Interface impedance ranging to 55 ohms is present in 21 of the 35 conduction sections; the cutoff sections show a maximum of 15 ohms in five of the 35 sections. Worthy of note is the fact that there is more interface impedance present on the conducting sections of this tungsten-nickel alloy cathode than on the cutoff sections, as is also the case on the 5687 tungsten-nickel alloy samples. This is just the reverse of experience on active alloys where silicon is the impurity.

4.1.2 Vacuum-Tube Research

An investigation of 30,000-hour 7AK7's from the five-digit multiplier has been completed and a report, M-3020, published. Considerable interelectrode leakage probably caused by sublimation on the micas from the cathode nickel sleeve was found. On tapping, transient decreases in interelectrode resistance probably caused by changing contact of the side rods with the micas resulted in more than half of the 30 tubes indicating intermittent shorts. The sensitivity of the detecting equipment is such that a maximum resistance of approximately 5 megohms lasting at least 500 microseconds can be detected Barring cathode problems, interelectrode leakage may be a serious limitation on long tube life.

An extensive investigation was made of the state of the cathodes of these tubes. To eliminate effects of geometry, the tubes were triode connected to +90 volts and the control grid pulsed at 1% duty factor up to +30 volts. A useful criterion of the state of the cathode which evolved from the investigation was the change in pulsed triode-connected plate current at a fixed high value of grid drive (+25 volts) while varying the heater voltage from 6.3 to 5.0 volts. This value is called "n" and is given as a percentage of the 6.3-v current. From the basic assumption that an increase in n is caused by an increase in the over-all work function of the cathode, it is possible to plot a family of curves showing variation in the triode-connected plate current as n varies; new tubes show an n of 6%.

By adding to the measured current of the old tubes the percentage decrease calculated from their n measurement, a distribution of currents is found whose mean is about 5% below that of new tubes and whose standard deviation is essentially the same as that of new

tubes. A Fisher "t" test, however, shows a fair probability that the 5% difference could be attributed to chance. Now by comparing the percent decrease from a perveance curve at a fixed triode-connected plate current (saturation) with the values predicted from the n calculation on several representative tubes, a correlation coefficient of 0.97 is found, indicating a rather high degree of certainty for the validity of the theoretical drop.

The n test has proven itself to be a rather valuable tool in our vacuum-tube work in determining whether a given low-current tube is a victim of a poisoned cathode or some other phenomenon.

4.2 Component Replacements

Fig. 4-3 lists the replacements of components other than tubes during the third quarter of 1954.

Component	Type	Total in Service	No. of Failures	Hours of Operation	Comments
Capacitor	Electrolytic 4.0-μf 450-v	136	1	4000 - 5000	Low capacitance
Crystal Diodes	1N34A (glass body)	16,881	2 2 1 2 21	2000 - 3000 5000 - 6000 6000 - 7000 7000 - 8000 8000 - 9000	Low R_b Low R_b Low R_b Low R_b Low R_b
	D-358 (ceramic body)	5137	1	25000 - 26000	Low R_b
	1N92 diffused junction	61	1	3000 - 4000	Drift
Rectifier	Selenium, IRC, 19PA 6, 156-v d-c, 5-ma	834	2	4000 - 5000	Open
Relay	Allied BODO-6D-38	675	1	1000 - 2000	Failed to operate
Resistor	Deposited carbon, 9000-ohm 1/2-watt $\pm$1%	878	1 1	0 - 1000 2000 - 3000	Above tolerance Above tolerance
Switch	Toggle, SPST	606	1	24000 - 25000	Intermittent contact
Trans-former	Pulse, Hipersil core, 3:1	4280	1	2000 - 3000	Open primary

Fig. 4-2. WWI Component Failures July 1 - September 30, 1954

5. ACADEMIC PROGRAM

5.1 MIT Courses

Summer Session, 1954

During the summer of 1954, MIT offered 35 different special summer programs, courses of 1 or 2 weeks duration largely for the benefit of interested people from business and industry. Two of these were directly concerned with digital computation, both being arranged by Professors Charles W. Adams and Stanley Gill with the full cooperation of the Digital Computer Laboratory. Several of the other programs were also of considerable interest in the field of computer application. For example, there were courses in Analogue Computation, Numerical Control of Machine Tools, Operations Research, and Mathematical Problems of Communications Theory.

The first of the digital-computer subjects, a 1-week seminar entitled Digital Computers: Advanced Coding Techniques, was intended as a meeting ground for experienced computer programmers interested in summarizing the present state of the art and in exchanging ideas for future work in automatic coding. Planned for 25 persons, the course was with great difficulty limited to 42, including many experts in the field. While concrete results are hard to summarize, it was felt to be a fruitful experience for all concerned.

An introductory 2-week course in Digital Computers: Business Applications stressed coding techniques and other aspects of computers in the light of commercial and industrial management information handling. The 61 students divided into four broad classes, with 25 representing 16 varied businesses, 17 representing 7 insurance companies, 7 representing a number of government agencies, and 12 representing 6 computer manufacturers and other special interests. To facilitate the teaching and use of practical computers typical of machines commercially available for business use, the summer-session computer developed for the 1953 summer session was revamped into a single-address computer (SAC), and an entirely new three-address computer (actually a 3 + 1 address machine), called TAC, was developed.

These two hypothetical computers proved very successful in demonstrating the various features of digital computers, with only a small attention to the many trivial, almost unavoidable details that tend to clutter up any practical computer design.

Fall Term, 1954

The principal course on machine computation being offered at MIT in the fall of 1954 is 6.25, Machine-Aided Analysis, a survey of computing techniques aimed largely at seniors in Electrical Engineering. This subject, first offered in the spring of 1954, had a fall-term enrollment of about 55 seniors and graduate students. It consequently was divided into three 20-man sections taught separately by Professors Linvill, Booton, and Adams. Practice problems were planned to allow each student to use both a REAC and the Whirlwind I computer (simulating the hypothetical TAC which had been developed for the 1954 summer session). Exercises using desk calculators and a card-programmed calculator, which were included in the first presentation of the subject, were eliminated in this second presentation to permit more time for studying techniques of problem analysis and of error analysis.

5.2 TAC--A Three-Address Computer

TAC is a hypothetical computer developed for use in MIT's special summer program in Digital Computers: Business Applications, August 1954. A compiler-interpreter program for MIT's Whirlwind I permits Whirlwind to simulate TAC.

TAC Definitions

A character is any of the following letters, digits, superscripts, symbols, or machine functions comprising the complete vocabulary of the MIT Flexowriters (augmented by \$ and ¢).

T C ¢ ¯ /) - + . 1 0 2 4 6 8 $^{0\ 2\ 4\ 6\ 8}$ a b...z

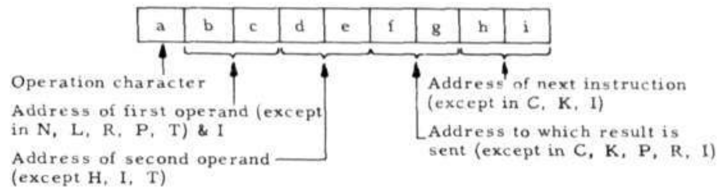
R B H \$: (_ = , | S 1 3 5 7 9 $^{1\ 3\ 5\ 7\ 9}$ A B...Z

where R = return carriage, T = tab (stops every tenth space), B = back space, C = color shift (starts black), H = stop, I = ignore, and S = space.

A word consists of nine characters, which are identified from left to right by the letters abcdefghi or by the digits 1 to 9.

A number is a word in which the first character, a, is -, +, or 0, and each of the remaining eight characters bcdefghi is one of nine digits 0, 1, 2, . . . , 9, forming an 8-digit decimal integer.

An instruction is a word in which the first character, a, represents one of the 15 operations listed below and the remaining characters, paired, form four "addresses." The scheme is shown below:



An address normally refers to one of 110 registers which are numbered 00, 01, . . . , 99, x0, x1, . . . , x9. However, there are certain exceptions to this:

1. The bc address associated with operations N, L, R, P, and T is omitted, and the characters are used as a continuation of the operation designation (e.g., N+, N-, below);

2. The fg address for operation P is omitted, and the characters are used to continue operation specification;

3. The registers x1 and x2 can be treated as a single register for certain purposes, and this register is denoted by the address xx. For operations A, S, M, D, or N, the contents of xx will be treated as a single 16-digit number, provided that x1 and x2 both contain 8-digit numbers having the same sign. Thus if x1 contains +12345678 and x2 contains +98765432, then xx will be deemed to contain +1234567898765432.

We note that address x0 always contains exactly zero. A post mortem (see below) will always occur if any instruction contains an address which is not legitimate according to the above rules.

A post mortem is performed automatically whenever TAC encounters any impossible instruction (in particular, any word not starting with one of 15 legitimate operation code letters, or containing an illegitimate address, or violating the special conditions listed on pages 66, 67, and 68). The post mortem consists of printing the location of the illegitimate instruction itself, the contents of the registers it refers to, a sequence table listing the locations of the instructions performed just prior to the post mortem, and an altered-word table listing the contents of all of the registers whose contents have been altered during the program.

The time required to perform a given instruction may be calculated from the following facts: TAC storage consists of a magnetic drum with four groups of 25 words (0-24, 25-49, 50-74, 75-99) revolving at 40 revolutions per second (1 millisecond per word); access to the x registers is one word time (no waiting time required); and TAC always follows the control sequence: it acquires the instruction, acquires the operand (s*), per-

* If two references to the drum are required to acquire operands or store results, TAC searches for both simultaneously, so that the time required is the longer of the two access times computed independently.

forms the required operation (in the number of word times indicated in the following list), and stores the result (s*) (omitting any storage accesses where none is required, as with the bc address of N, L, R, P, or T).

Symbols Used

abcdefghi represents the nine characters of a TAC word.

C( ) represents "contents of." Thus C(bc) represents "the contents of the register whose address is bc" (where b and c, the second and third characters of the instruction, must each be either one of the decimal digits or the letter x).

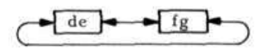
→ represents "becomes the new contents of." Consequently: C(bc) + C(de)→fg should be read as "the contents of bc plus the contents of de becomes the new contents of register fg, replacing whatever was in fg but not changing what is in bc or de."

xx represents TAC's double-length register made by pairing x1 and x2.

> represents "is greater than." < represents "is less than."

10^c (read as "ten to the c") represents a one followed by c zeros (for example, 10^5 = 100, 000).

TAC Instruction Code

Name	Code Letter	Word Times /Operation	Function	Post Mortem Will Occur
Read	R	5/char.	Read enough characters from punched tape to fill the positions numbered b thru c in register de, without changing the other digits of C(de). (ignores deletions, deals properly with upper and lower case, but reads all other characters including back spaces, underlines, tabs, and carriage returns explicitly, and $ and ¢ are each read as three characters)	If tape contains illegal characters, or if b = 0, or if c = 0, or if b > c, or if de = xx or is not a legitimate address.
Print	P	10/char.	Print the characters in the positions numbered b thru c in register de, preceded by the character f and followed by g.	Unless b = 1, 2, . . . , 9 and c = 1, 2, . . . , 9 and de is a legitimate address (not xx) or if b > c.
Tape Read	Tr	30/block	Read ten words from block de (or from next consecutive block if de = 00) on tape unit c into registers fg, fg + 1, . . . , fg + 9.	Unless b = w or r " c = 1, 2, 3, 4 and " fg = 00, 01, . . . , 90 or x0 " de = 00, 01, . . . , or 40 if de = 00 and block 40 has been just read or written.
Tape Write	Tw		Write ten words onto block de (or onto next consecutive block if de = 00) on unit c from registers fg, fg + 1, . . . , fg + 9.	
Input	I	5/char.	Without altering the present contents of storage, start reading a punched tape containing a TAC program in conventional form, ending by taking the next instruction from the address preceding "start" at the end of the tape.	
Halt	H		Stop computing. Start at hi only if the start button is depressed.	
Add	A	2	C(bc) + C(de)→fg	1.* If the magnitude of the result exceeds 99,999, 999 (or 9, 999, 999, 999, 999, 999 if fg = xx); 2. If bc = xx or de = xx or fg = xx and (bc) and (de) are not both numbers. 3. If the column-by-column addition or subtraction involves any nondigit N in any arithmetic operation other than N + 0 = N, N - 0 = N, or N - N = 0 (see next page for details).
Subtract	S	2	C(bc) - C(de)→fg	
Multiply	M	10	C(bc) x C(de)→fg	1.* If magnitude of result exceeds 99, 999, 999 (or 9,999, 999, 999, 999, 999 if fg = xx); 2a. In M and D only, unless C(bc) and C(de) are both numbers; 2b. In N only, unless b = + or -, c = decimal digit, and C(de) = a number.
Divide	D	25	C(bc) ÷ C(de)→fg (quotient rounded)	
Numerical Shift Left	N+	2	C(de) x 10^c→fg	
Shift Right	N-	2	C(de) ÷ 10^c→fg	
Compare Numerically	C	2	Take next instruction from: fg if C(bc) > C(de), hi if C(bc) < C(de), or next register consecutively if C(bc) = C(de).	* If C(bc) and C(de) are not both numbers.
Logical Shift	L	2	Shift C(de) and C(fg) cyclically c places (left if b is +, right if b is -). ←b = + b = - → 	Unless b = + or -; c = 0, 1, . . . , or 9; and de and fg are legitimate addresses; or if bc, de, or fg = xx.

* A post mortem occurs on A, S, M, D, C, K, and E unless bc, de, and fg are all legitimate addresses.

Compare Logically	K	2	Take next instruction from fg if C(bc)>C(de), hi if C(bc)<C(de), or next register consecutively if C(bc) is identical with C(de). Here the symbol > is defined as follows: Compare the characters of C(bc) with those of C(de) column by column from the left end until identify is established or until one character is found to be to the right of the other in the list below**, in which case the word containing said character is said to be greater than the other logically.	* If bc, de, or fg = xx.
Extract	E	2	In those columns, and only those, in which C(de) has odd characters (listed in the lower line of the two lines below**), replace the characters of C(fg) by the characters occupying corresponding columns in C(bc), without altering the other characters in C(fg).	* If bc, de, or fg = xx.

** small end { even: T C ¢ ‾ /) - + . I 0 2 4 6 8 $^{0\ 2\ 4\ 6\ 8}$ a b...z ; odd: R B H $: (_ = , | S 1 3 5 7 9 $^{1\ 3\ 5\ 7\ 9}$ A B...Z } large end

1. Register x0 contains 000000000. If an instruction attempts to put other information into it, the information is lost. No post mortem occurs.

2. The next instruction is taken from hi (unless otherwise specified in operations C and K) except in I.

3. A post mortem always occurs if hi is not a legitimate address (except in I, and in H if not restarted).

Addition and Subtraction in TAC

The rules which TAC obeys in adding or subtracting are the following:

1. While a positive number may start with zero instead of plus, a positive sum or difference formed by TAC always starts with a plus sign.

2. If any or all of the addresses bc, de, or fg in the instruction designate register xx, both C(bc) and C(de) must be numbers. If fg does not equal xx, the magnitude of the sum or difference must be less than 10^8.

3. If none of the addresses bc, de, or fg designates register xx, the situation depends on whether addition or subtraction is required and on what the first characters of C(bc) and C(de) are. Summarized below are all possible combinations of such first characters. The symbols used are:

θ represents either a plus sign or a zero

± represents a plus sign or a minus sign as appropriate

k represents any character other than +, -, or 0

k' represents any other character other than +, -, or 0 (different from k)

... represents any set of eight characters to the right of the sign, i.e., in positions bc2-9 or de2-9

(w) - letters in parentheses refer to the first of three different cases tabulated at the bottom of the page, wherein the right-hand eight digits of the results are discussed in terms of C(bc2-9) and C(de2-9)

P.M. indicates that no result is possible and that a post mortem always results in this situation

	if C(bc) =	θ...	θ...	θ...	-...	-...	-...	k...	k...	k...	k...
for	and C(de) =	θ...	-...	k...	θ...	-...	k...	θ...	-...	k...	k'...
ADD	then C(fg) =	θ(w)	±(x)	k(w)	±(x)	-(w)	k(y)	k(w)	k(y)	P.M.	P.M.

	if C(bc) =	θ...	θ...	θ...	-...	-...	-...	k...	k...	k...	k...
for	and C(de) =	θ...	-...	k...	θ...	-...	k...	θ...	-...	k...	k'...
SUBTRACT	then C(fg) =	±(x)	+(w)	P.M.	-(w)	±(x)	P.M.	k(y)	k(w)	±(x)	P.M.

	The right-hand eight columns of the result, i.e., C(fg2-9), are derived from C(bc2-9) and C(de2-9) thus:	A post mortem will occur
w	C(bc2-9) and C(de2-9) are simply added column by column.	IF the sum exceeds 99,999,999 or IF the addition requires adding a carry or any character other than zero to any nondigit.
x	If C(bc2-9) is larger than C(de2-9), assuming all nondigits to be larger than zero, then C(de2-9) is subtracted column by column from C(bc2-9), otherwise C(bc2-9) is subtracted column by column from C(de2-9).	IF the subtraction requires subtracting a borrow or any character other than zero or itself from any nondigit.
y	The quantity C(bc2-9), or C(de2-9), whichever is preceded by the minus sign, is subtracted column by column from C(de2-9) or C(bc2-9).	IF the difference so formed is negative or IF the condition of case x arises.

5.3 Visitors

Tours of the WWI installation include a showing of the film "Making Electrons Count," a computer demonstration, and an informal discussion of the major computer components. During the past quarter 15 groups visited the computer installation. Included in these groups were:

July 7	Special Foreign Students, MIT
August 3	MIT Summer Session Course -- Digital Computers: Advanced Coding Techniques
August 4	Secondary School Science Teachers
August 18-21	MIT Summer Session Course -- Digital Computers: Business Applications
August 24	Teachers Group
September 22	Worcester Polytech Students

6. APPENDIX

6.1 Publications

Project Whirlwind technical reports and memorandums are routinely distributed to only a restricted group known to have a particular interest in the Project, and to ASTIA (Armed Services Technical Information Agency) Document Service Center, Knott Building, Dayton, Ohio. Requests for copies of individual reports should be made to ASTIA.

The following is a list of memoranda published by the Scientific and Engineering Computations Group during the past quarter:

No.	Title	Date	Author
SR-38	Summary Report No. 38, Second Quarter 1954		
M-2891	Proposed Change in WWI to Make Possible Programmed Recording of the "Lights"	7-13-54	A. Siegel
DCL-7	The "FL" Flexowriter Code, Binary Numerical Sequence - reprint p. 20, M-1623		
DCL-8	The "FL" Flexowriter Code, Alphanumerical Sequence - reprint p. 19, M-1623		
DCL-11	Automatic Scope Output Display Routines	8-30-54	N. J. Saber
DCL-12	A Program for Transferring Binary Information Back and Forth between Paper Tape and Magnetic Tape	9-1-54	S. F. Best